

ปริภูมิควอไซจิเนอรัลไลซ์วิคซิม: ปริภูมิ- $q_s$   
Quasi Generalized Weak Sum Space:  $q_s$ -Space

สัจจารักษ์ ลาดสูงเนิน  
Sajjarak Ladsungnern

สาขาวิชาคณิตศาสตร์ คณะครุศาสตร์ มหาวิทยาลัยราชภัฏชัยภูมิ จังหวัดชัยภูมิ 36000  
Department of Mathematics, Faculty of Education, Chaiyaphum Rajabhat University,  
Chaiyaphum 36000, Thailand  
Corresponding Author, E-mail: Ladsungnern@gmail.com

Received: 11 June 2021, Revised: 12 November 2021, Accepted: 15 December 2021

บทคัดย่อ

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อแนะนำปริภูมิควอไซจิเนอรัลไลซ์วิคซิม (เรียกสั้น ๆ ว่าปริภูมิ- $q_s$ ) ซึ่งเป็นการขยายจากปริภูมิควอไซจิเนอรัลไลซ์วิคแล้วทำการศึกษาสมบัติเชิงคุณภาพพื้นฐานภายในปริภูมิ- $q_s$  นี้ในรูปของเซตภายใน- $q_s$  เซตส่วนปิดคลุม- $q_s$  และเซตแก่นกลาง- $q_s$  ที่ได้นิยามขึ้นมา ซึ่งจากการศึกษาพบว่า การดำเนินการพื้นฐานของเซตเหล่านี้ทำให้ได้สมบัติต่างๆภายใต้การดำเนินการนั้น ๆ ในแต่ละอย่างที่แตกต่างกันไปและยิ่งกว่านั้นเราจะพบว่าไม่มีเพียงเซตภายใน- $q_s$  เท่านั้นที่บรรจุในยูเนียนใด ๆ ของการอินเตอร์เซกของเซต ภายใน- $q_i$  ของเซต  $A$  ใด ๆ กับเซต  $X_i$  บนปริภูมิ  $(X_i, q_i)$ , ทุก  $i \in I$  ในขณะที่เซตส่วนปิดคลุม- $q_s$  และเซตแก่นกลาง- $q_s$  นั้นจะบรรจุในยูเนียนใด ๆ ของการอินเตอร์เซกของเซตส่วนปิดคลุม- $q_i$  (เซตแก่นกลาง- $q_i$  ตามลำดับ) ของเซต  $A$  ใด ๆ กับเซต  $X_i$  บนปริภูมิ  $(X_i, q_i)$ , ทุก  $i \in I$

คำสำคัญ : ปริภูมิ- $q_s$  เซตเปิด- $q_s$  เซตปิด- $q_s$  เซตภายใน- $q_s$  เซตส่วนปิดคลุม- $q_s$  และเซตแก่นกลาง- $q_s$

Abstract

The purpose of this research was to introduce a generalized of quasi generalized weak space called quasi generalized weak sum space (as Known briefly as,  $q_s$ -space), and to study the intrinsic qualitative fundamental properties in term of  $q_s$ -interior,  $q_s$ -closure and  $q_s$ -kernel. The results found that these sets are working on  $q_s$ -space and gives various fundamental properties under defined by the definition of them and basic operations on sets on  $q_s$ -space. Furthermore, it was found that only the  $q_s$ -interior set is contained in the arbitrary union of intersection of  $q_i$ -interior for any set  $A$  and  $X_i$  on the space  $(X_i, q_i)$ ,  $\forall i \in I$ , but contrary for  $q_s$ -closure, and  $q_s$ -kernel. They demonstrates the union of

intersection of  $q_i$ -closure and  $q_i$ -kernel for any set  $A$  and  $X_i$  respectively, on the space  $(X_i, q_i), \forall i \in I$ .

**Keywords:**  $q_s$ -space,  $q_s$ -open,  $q_s$ -closed,  $q_s$ -interior,  $q_s$ -closure,  $q_s$ -kernel

## 1. Introduction

The extension of the space was very important way to generalized concept for researcher to study and investigate especially topological space. Zvina (Zvina, 2011) introduced the notion of generalized topological space (gt-space). Generalized topology of gt-space has the structure of frame and is closed under arbitrary union and finite intersection modulo subsets. The family of small subsets of a gt-space forms an ideal that is compatible with the generalized topology. Császár (Császár, 2012) was the first who introduced the notion of generalized topological space called the weak structure. After that, Ávila (Ávila, 2012) extended the weak structure to generalized weak structure briefly, GWS. Recently, Janrongkam (Janrongkam, 2019) extend GWS to quasi generalized weak structure briefly. QGWS and Thongpan (Thongpan, 2019) extend the last space to bi-quasi generalized weak space.

In this paper, we introduced the concepts of generalized of quasi generalized weak structure namely quasi generalized weak sum space briefly,  $q_s$ -space and to study some fundamental of qualitative intrinsic properties on this space.

## 2. Preliminaries

In this section, we presented some basic concepts of quasi generalized weak space. All results were presented by Janrongkam (2019).

**Definition 2.1** A quasi generalized weak structure (briefly QGWS) on the nonempty set  $X$ , is a nonempty collection  $q$  of subsets of  $X$  satisfying the property:  $U \cap V \in q$  for any  $U, V \in q$ . A quasi generalized weak space consists of two objects: a nonempty set  $X$  and a quasi-generalized weak structure  $q$  on  $X$  and is denoted by  $(X, q)$ . Each member of  $q$  is called a  $q$ -open set in  $X$  (briefly  $q$ -open set) and the complement of  $q$ -open set is said to be  $q$ -closed set in  $X$  (briefly,  $q$ -closed set).

It is important to note that each topology is a QGWS, and each QGWS is a GWS as shown in the diagram below: topology  $\Rightarrow$  QGWS  $\Rightarrow$  GWS.

**Definition 2.2** Let  $q$  be a QGWS on  $X$  and  $A \subset X$ .

1. The  $q$ -interior of  $A$ . denoted by  $i_q(A)$  and is defined by

$$i_q(A) = \cup \{U : U \in q \text{ and } U \subset A\},$$

2. The  $q$ -closure of  $A$ , denoted by  $c_q(A)$ , and is defined by

$$c_q(A) = \bigcap \{F : F \in q \text{ is } q\text{-closed and } A \subset F\}.$$

Each element of  $i_q(A)$  is said to be a  $q$ -interior (respectively,  $q$ -adherent) point of  $A$ .

### 3. Main Results

In this section, we shall be present about a generalized of quasi generalized weak spaces, called quasi generalized weak sum space and study to its qualitative intrinsic properties.

#### 3.1 Quasi Generalized Weak Sum Space

First of all, we shall give an important theorem as its useful for our work.

**Theorem 3.1.3** Let  $\{(X_i, q_s) : i \in I\}$  be any collection of pairwise disjoint quasi generalized weak spaces. Let  $\bigoplus_{i \in I} X_i = \bigcup_{i \in I} X_i$  and  $q_s$  be a collection of subset of  $\bigoplus_{i \in I} X_i$  defined as follow:

$$q_s = \left\{ U : U \subseteq \bigoplus_{i \in I} X_i \text{ and } U \cap X_i \in q_i \right\}.$$

Then  $q_s$  is a quasi-generalized weak structure for  $\bigoplus_{i \in I} X_i$ .  
**Proof** Let  $U, V \in q_s$ , we shall show that  $q_s$  is a quasi-generalized weak structure for  $\bigoplus_{i \in I} X_i$ . Since  $U, V \in q_s$ , we have  $U \cap X_i \in q_i$  and  $V \cap X_i \in q_i$  for all  $i \in I$ . Hence, so we have  $U \cap V \in q_s$ . Therefore  $q_s$  is a quasi-generalized weak structure for  $\bigoplus_{i \in I} X_i$ .

**Definition 3.1.4** By Theorem 3.1.1, the pair  $(\bigoplus_{i \in I} X_i, q_s)$  is called the quasi generalized weak sum space of the space  $\{(X_i, q_s) : i \in I\}$  and briefly  $q_s$ -space. Each element of  $q_s$  is said to be  $q_s$ -open and the complement of  $q_s$ -open is called  $q_s$ -closed sets.

**Example 3.1.5** Let  $I = \{1, 2\}$ ,  $X_1 = \{a, b\}$ ,  $q_1 = \{\{a\}, \{a, b\}\}$ .

$$X_2 = \{c, d, e\}, \quad q_2 = \{\{c\}, \{c, d\}, \{c, d, e\}\}.$$

We obtained  $\bigoplus_{i \in I} X_i = \{a, b, c, d, e\}$  and

$$q_s = \{\{a, c\}, \{a, b, c\}, \{a, c, d\}, \{a, c, d, e\}, \{a, b, c, d\}, \{a, b, c, d, e\}\}.$$

The  $q_s$ -closed are:  $\emptyset, \{e\}, \{b\}, \{b, e\}, \{d, e\}, \{b, d, e\}$ .

Throughout this paper, there are the symbols for use as follow:

1.  $(\bigoplus_{i \in I} X_i, q_s)$  denoted by a  $q_s$ -space of the space  $\{(X_i, q_s) : i \in I\}$ ,
2.  $q_s\text{-}o(\bigoplus_{i \in I} X_i)$  denoted by the collection of all  $q_s$ -open sets on  $\bigoplus_{i \in I} X_i$ ,



3.  $q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right)$  denoted by the collection of all  $q_s\text{-}$  closed sets on  $\bigoplus_{i \in I} X_i$ .

**Theorem 3.1.6** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s\text{-}$ space, and  $A \subseteq \bigoplus_{i \in I} X_i$  be a  $q_s\text{-}$ open set if

$$A = \bigcup_{i \in I} (A \cap X_i).$$

**Proof** Let  $A \in q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right)$  we want to show that  $A = \bigcup_{i \in I} (A \cap X_i)$ .

Since  $A \cap X_i \subseteq A$ , so that  $\bigcup_{i \in I} (A \cap X_i) \subseteq A$ . ... (1)

Since  $A \in q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right)$ , we have  $A \cap X_i \in q_i \forall i \in I$ .

That is,  $x \in A \Rightarrow x \in A \cap X_i \forall i \in I$ .

$$\Rightarrow x \in A \cap X_i \exists i \in I.$$

$$\text{So } A \subseteq \bigcup_{i \in I} (A \cap X_i). \quad . (2)$$

By (1), (2) we obtained  $A = \bigcup_{i \in I} (A \cap X_i)$ .

Conversely, let  $A = \bigcup_{i \in I} (A \cap X_i)$  we want to show that  $A$  is  $q_s\text{-}$  open set.

By Definition 3.1.4,  $A \cap X_i \in q_i \forall i \in I \Leftrightarrow A \in q_s$ . Therefore  $\bigcup_{i \in I} (A \cap X_i) = A \in q_s$ . #.

**Note 3.1.7** From Definition 3.1.4 and Theorem 3.1.6. We have :

If  $G \in q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right)$  then  $G = \bigcup_{i \in I} (G_i)$  when  $G_i \in q_i\text{-}O(X_i) i \in I$ .

If  $F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right)$  then  $F = \bigcap_{i \in I} (F_i)$  when  $F_i \in q_i\text{-}C(X_i), i \in I$ .

**Theorem 3.1.8** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s\text{-}$ space, and  $A, B$  are  $q_s\text{-}$ open sets. Then

1.  $A \cup B$  is a  $q_s\text{-}$ open set,

2.  $A \cap B$  is a  $q_s\text{-}$ open set.

**Proof :** 1. Since  $A, B$  are  $q_s\text{-}$ open sets.

We have,  $A \subseteq \bigoplus_{i \in I} X_i$  and  $A \cap X_i \in q_i \forall i \in I$ ,

$$B \subseteq \bigoplus_{i \in I} X_i \text{ and } B \cap X_i \in q_i \forall i \in I.$$

Then  $A \cup B \subseteq \bigoplus_{i \in I} X_i$  and  $(A \cap X_i) \cup (B \cap X_i) \in q_i$ .

$$\Rightarrow [(A \cup B) \cap X_i] \in q_i.$$

Therefore  $A \cup B$  is a  $q_s\text{-}$ open set. #.

2. similarly 1. #.

**Remark 3.1.9** The converse of Theorem 3.1.8 1. not be true, see Example 3.1.5.

Let  $A = \{a\}$ ,  $B = \{c\}$ , we have  $A \cup B = \{a, c\}$  is  $q_s\text{-}$  open, but  $A, B$  are not  $q_s\text{-}$ open sets.

**Theorem 3.1.10** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space, and  $A, B$  are  $q_s$ -closed sets.

- Then
1.  $A \cup B$  is  $q_s$ -closed set,
  2.  $A \cap B$  is  $q_s$ -closed set.

**Proof** Since  $A, B$  are  $q_s$ -closed sets, so we have  $A^c, B^c$  are  $q_s$ -open sets. Since  $(A \cup B)^c = A^c \cap B^c$  and  $(A \cap B)^c = A^c \cup B^c$  and by Theorem 3.1.8, we obtained the results.

**Remark 3.1.11** For a  $q_s$ -space  $\left(\bigoplus_{i \in I} X_i, q_s\right)$ , then  $\emptyset, \bigoplus_{i \in I} X_i$  be not necessary  $q_s$ -open or  $q_s$ -closed sets.

### 3.2 $q_s$ -Interior

**Definition 3.2.12** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space and  $A \subseteq \bigoplus_{i \in I} X_i$ . The set of all  $q_s$ -interior of  $A$  denoted by  $i_{q_s}(A)$  defined by

$$i_{q_s}(A) = \bigcup \left\{ U \in q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right) : U \subseteq A \right\}.$$

In other word, a point  $x \in A$  is said to be  $q_s$ -interior point of  $A$  if there exist are  $q_s$ -open set  $U$  on  $\bigoplus_{i \in I} X_i$  such that  $x \in U \subseteq A$ .

**Example 3.2.13** Let  $I = \{1, 2, 3\}$ ,  $X_1 = \{a, b\}$ ,  $X_2 = \{c, d\}$ ,  $X_3 = \{e\}$ ,  
 $q_1 = \{\{a\}, \{a, b\}\}$ ,  $q_2 = \{\{d\}, \{c, d\}\}$ ,  $q_3 = \{\{e\}\}$ . We have  $\bigoplus_{i \in I} X_i = \{a, b, c, d, e\}$  and  
 $q_s = \{\{a, d, e\}, \{a, c, d, e\}, \{a, b, d, e\}, \{a, b, c, d, e\}\}$ .  
 $= q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right)$ . Then  $q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right)$  are  $\{c, b\}, \{b\}, \{c\}, \emptyset$ . Let  $A = \{a, b\}$ , then  
 $i_{q_s}(A) = \bigcup \{ \} = \emptyset$ .

**Note 3.2.14** By Definition 3.2.12, let  $A \in X_i, i \in I$ . Then

1.  $i_{q_i}(A) = \bigcup \{ V \in q_i\text{-}O(X_i) : V \subseteq A \}$ ,
2.  $i_{q_s}(A) \subseteq \bigcup_{i \in I} i_{q_i}(A)$ ,
3.  $A \in q_i\text{-}O(X_i)$  then  $i_{q_i}(A) = A$ .

**Example 3.2.15** From Example 3.2.13, let  $A = \{a, b\} \subseteq X_1$ , we have

$$\begin{aligned} i_{q_1}(A) &= \bigcup \{ V \in q_1\text{-}O(X_1) : V \subseteq A \} \\ &= \bigcup \{ \{a, b\} \} = \{a, b\}. \end{aligned}$$

But  $i_{q_s}(A) = \emptyset$  so we have  $i_{q_s}(A) = \bigcup_{i \in I} i_{q_i}(A)$ .

Since  $A = \{a, b\} \in q_1\text{-}O(X_1)$ , then by (1)  $i_{q_s}(A) = \{a, b\} = A$ .



In this following, we discuss to some properties of  $q_s$ -interior of any set on  $q_s$ -space.

**Theorem 3.2.16** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space and  $A, B \subseteq \bigoplus_{i \in I} X_i$ . Then

1.  $i_{q_s}(\emptyset) = \emptyset$ ,
2.  $i_{q_s}(\bigoplus_{i \in I} X_i) \subseteq \bigoplus_{i \in I} X_i$ ,
3.  $i_{q_s}(A) = A$ ,
4. If  $A \subseteq B$  then  $i_{q_s}(A) \subseteq i_{q_s}(B)$ ,
5.  $i_{q_s}(A)$  is a  $q_s$ -open set,
6.  $i_{q_s}(A)$  is the largest  $q_s$ -open set contained in  $A$ .

**Proof** 1. Let  $x \in i_{q_s}(\emptyset) \Leftrightarrow \exists U \in q_s-o(\bigoplus_{i \in I} X_i)$  such that  $x \in U \subseteq \emptyset \dots (*)$

Right side (\*) is true, when  $U = \emptyset$ , so we have  $i_{q_s}(\emptyset) = \emptyset$ . #.

2. and 3., holds from Definition 3.2.12. #.

4. Let  $A \subseteq B$ , we have

$$\cup \left\{ U \in q_s-o\left(\bigoplus_{i \in I} X_i\right) : U \subseteq A \right\} \subseteq \cup \left\{ U \in q_s-o\left(\bigoplus_{i \in I} X_i\right) : U \subseteq B \right\}$$

Therefore  $i_{q_s}(A) \subseteq i_{q_s}(B)$ . #.

5. By Definition 3.2.12. #.

6. From 5, we have  $i_{q_s}(A)$  be  $q_s$ -open sets, and  $i_{q_s}(A) \subseteq A$ . Let  $U \subseteq A$

be  $q_s$ -open set, we want to show that  $U \subseteq i_{q_s}(A)$  From Definition 3.2.12, we result desired.

**Remark 3.2.17** From Theorem 3.2.16 2., be not necessary to equal of two side. See Example 3.2.18.

**Example 3.2.18** Let  $I = \{1, 2\}$ ,  $X_1 = \{a, b\}$ ,  $X_2 = \{c, d\}$ ,  $q_1 = \{\{a\}\}$ ,  $q_2 = \{\{c\}, \{c, d\}\}$ .

We have  $\bigoplus_{i \in I} X_i = \{a, b, c, d\}$ ,

$$q_s = \{\{a, c\}, \{a, c, d\}\} = q_s-o\left(\bigoplus_{i \in I} X_i\right),$$

$$i_{q_s}\left(\bigoplus_{i \in I} X_i\right) = \cup \left\{ U \in q_s-o\left(\bigoplus_{i \in I} X_i\right) : U \subseteq \bigoplus_{i \in I} X_i \right\}.$$

$$= \cup \{\{a, c\}, \{a, c, d\}\}.$$

$$= \{a, c, d\} \neq \bigoplus_{i \in I} X_i.$$

**Theorem 3.2.19** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space, and  $A \subseteq \bigoplus_{i \in I} X_i$ . Then

1.  $A \in q_s-o\left(\bigoplus_{i \in I} X_i\right)$  if and only if  $A = i_{q_s}(A)$ ,
2.  $i_{q_s}(i_{q_s}(A)) = i_{q_s}(A)$ .



**Proof** 1. By Theorem 3.2.16, we have  $i_{q_S}(A) \subseteq A$ , since  $A \in q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right)$ , and  $A \subseteq A$ . By Definition 3.2.12, let  $x \in A \Rightarrow x \in i_{q_S}(A)$ , hence  $A \subseteq i_{q_S}(A)$ .

Therefore  $A = i_{q_S}(A)$ .

Conversely, let  $A = i_{q_S}(A)$ .

$$= \cup \left\{ U \in q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right) : U \subseteq A \right\}.$$

So, we have  $A = U$  is  $q_S$ -open set. #.

2. Since  $i_{q_S}(A) \in q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right)$  and by (1) we get as desired. #.

**Theorem 3.2.20** Let  $\left(\bigoplus_{i \in I} X_i, q_S\right)$  be a  $q_S$ -space,  $A \subseteq \bigoplus_{i \in I} X_i$ . Then

$$i_{q_S}(A) \subseteq \bigcup_{i \in I} i_{q_i}(A \cap X_i).$$

**Proof** Let  $A \subseteq \bigoplus_{i \in I} X_i$ , and let  $x \in i_{q_S}(A) \Rightarrow x \in U \subseteq A$  where  $U \in q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right)$ .

$$\Rightarrow x \in U \cap X_i \subseteq A \cap X_i, U \cap X_i \in q_i, i \in I.$$

$$\Rightarrow x \in i_{q_i}(A \cap X_i), i \in I.$$

$$\Rightarrow x \in \bigcup_{i \in I} i_{q_i}(A \cap X_i).$$

Hence,  $i_{q_S}(A) \subseteq \bigcup_{i \in I} i_{q_i}(A \cap X_i)$ . #.

**Example 3.2.21** Let  $I = \{1, 2\}$ ,  $X_1 = \{a, b\}$ ,  $X_2 = \{c, d\}$ ,  $\bigoplus_{i \in I} X_i = \{a, b, c, d\}$

$$q_1 = \{\{a\}, \{a, b\}\}, q_2 = \{\{d\}, \{c, d\}\},$$

$$q_S = \{\{a, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c, d\}\}$$

$$= q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right).$$

Let  $A = \{c, d\}$ , we have  $i_{q_S}(A) = \emptyset$  and  $\bigcup_{i \in I} i_{q_i}(A \cap X_i) = \{c, d\}$ .

**Remark 3.2.22** From Theorem 3.2.20, if  $A \in q_S\text{-}o\left(\bigoplus_{i \in I} X_i\right)$ ,

then  $i_{q_S}(A) = \bigcup_{i \in I} i_{q_i}(A \cap X_i)$ . Since for  $i \in I$ ,

$$A \cap X_i \subseteq A \Rightarrow \bigcup_{i \in I} (A \cap X_i) \subseteq A = i_{q_S}(A).$$

$$\text{So, we have } i_{q_S}(A) = \bigcup_{i \in I} i_{q_i}(A \cap X_i).$$

**Theorem 3.2.23** Let  $\left(\bigoplus_{i \in I} X_i, q_S\right)$  be a  $q_S$ -space, and  $A, B \subseteq \bigoplus_{i \in I} X_i$ . Then

$$1. i_{q_S}(A) \cup i_{q_S}(B) \subseteq i_{q_S}(A \cup B),$$

$$2. i_{q_S}(A \cap B) = i_{q_S}(A) \cap i_{q_S}(B).$$

**Proof:** 1. Since  $A \subseteq A \cup B$ ,  $B \subseteq A \cup B$  by Theorem 3.2.16 4., we have



$$i_{q_s}(A) \subseteq i_{q_s}(A \cup B) \text{ and } i_{q_s}(B) \subseteq i_{q_s}(A \cup B).$$

$$\text{Hence, } i_{q_s}(A) \cup i_{q_s}(B) \subseteq i_{q_s}(A \cup B).$$

#.

2. Since  $A \cup B \subseteq A$  and  $A \cup B \subseteq B$ .

$$\text{We have } i_{q_s}(A \cap B) \subseteq i_{q_s}(A) \cap i_{q_s}(B). \quad \dots (*)$$

By Theorem 3.2.16 6. and  $i_{q_s}(A), i_{q_s}(B)$  are  $q_s$ -open sets, we have

$$i_{q_s}(A) \cap i_{q_s}(B) \subseteq i_{q_s}(A \cap B). \quad \dots (**)$$

$$\text{By } (*), (**), \text{ we have } i_{q_s}(A \cap B) = i_{q_s}(A) \cap i_{q_s}(B).$$

#.

**Remark 3.2.24** On Theorem 3.2.23 1. not necessary equal to two side, see Example 3.2.13, Let  $A = \{a, d\}, B = \{e\}$ . We have  $i_{q_s}(A) = \emptyset = i_{q_s}(B) = i_{q_s}(A) \cup i_{q_s}(B) = \emptyset$ , but  $i_{q_s}(A \cup B) = i_{q_s}\{a, d, e\} = \{a, d, e\}$ .

### 3.3 $q_s$ -closure

**Definition 3.3.25** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space, and  $A \subseteq \bigoplus_{i \in I} X_i$ . The  $q_s$ -closure of  $A$ , denoted by  $c_{q_s}(A)$ , is defined by  $c_{q_s}(A) = \bigcap \{F \in q_s\text{-}C(\bigoplus_{i \in I} X_i) : A \subseteq F\}$ .

**Note 3.3.26** By Definition 3.3.25, we see that

$$c_{q_i}(A) = \bigcap \{F \in q_i\text{-}C(X_i), i \in I : A \subseteq F\} \subseteq c_{q_s}(A). \text{ By Note 3.1.7,}$$

$$\text{we have } F \in q_i\text{-}C(\bigoplus_{i \in I} X_i), F = \bigcup_{i \in I} F_i \text{ where } F_i \in q_i\text{-}C(X_i), i \in I, \text{ so that } \bigcup_{i \in I} c_{q_i}(A) \subseteq c_{q_s}(A).$$

**Example 3.3.27** See Example 3.2.13, let  $A = \{a, b\} \subseteq X_1$ . We have

$$c_{q_1}(A) = \bigcap \{ \} = \{a, b\} \text{ and } c_{q_s}(A) = \bigcap \{ \} = \{a, b, c, d, e\}.$$

hence  $c_{q_1}(A) \subseteq c_{q_s}(A)$ . In this following, we discuss to some properties of  $q_s$ -

closure of any set on  $q_s$ -space.

**Theorem 3.3.28** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space, and  $A, B \subseteq \bigoplus_{i \in I} X_i$ . Then

1.  $c_{q_s}(\bigoplus_{i \in I} X_i) \subseteq \bigoplus_{i \in I} X_i$ ,
2.  $c_{q_s}(A) = A$ ,
3. If  $A \subseteq B$  then  $c_{q_s}(A) \subseteq c_{q_s}(B)$ ,
4.  $c_{q_s}(A)$  is a  $q_s$ -closed set,
5.  $c_{q_s}(A)$  is the smallest  $q_s$ -closed set containing  $A$ , and
6.  $A \in q_s\text{-}C(\bigoplus_{i \in I} X_i)$  if and only if  $c_{q_s}(A) = A$ .



**Proof 1.** By Definition 3.3.25,

$$\text{we have } c_{qs} \left( \bigoplus_{i \in I} X_i \right) \subseteq \left\{ F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : \bigoplus_{i \in I} X_i \subseteq F \right\},$$

this statement is true when  $F \subseteq \bigoplus_{i \in I} X_i$ .

$$\text{Hence } c_{qs} \left( \bigoplus_{i \in I} X_i \right) = \bigcap \left\{ \bigoplus_{i \in I} X_i \right\} = \bigoplus_{i \in I} X_i. \quad \#$$

2. let  $x \in A$ , we want to show that  $x \in c_{qs}(A)$ . By definition of  $q_s$ -closure of  $A$ , if  $x \in A$  then  $x \in F$ , therefore  $x \in \bigcap \left\{ F : F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\} = c_{qs}(A)$ . Hence  $A \subseteq c_{qs}(A)$ .  $\#$

3. If  $A \subseteq B$ , by Definition 3.3.25, we have

$$\bigcap \left\{ F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\} \subseteq \left\{ F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : B \subseteq F \right\}.$$

Therefore  $c_{qs}(A) \subseteq c_{qs}(B)$ .  $\#$

4. Since  $\left\{ F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\}$  be a  $q_s$ -closed set, by

Theorem 3.1.10 we have  $\bigcap \left\{ F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\} = c_{qs}(A)$  is closed.  $\#$

5. From 2., 4., we have  $A \subseteq c_{qs}(A)$ . Let  $F \in q_s\text{-}C \left( \bigoplus_{i \in I} X_i \right)$  such that  $A \subseteq F$ . We shall show that  $c_{qs}(A) \subseteq F$ . Let  $x \in c_{qs}(A)$  by Definition 3.3.25, we have  $x \in F$ . Hence  $c_{qs}(A) \subseteq F$ , therefore  $c_{qs}(A)$  is the smallest  $q_s$ -closed set containing  $A$ .  $\#$

6. Let  $A$  be  $q_s$ -closed set, we want to prove that  $A = c_{qs}(A)$ . By Definition 3.3.25, we let give  $A = F$ , so  $c_{qs}(A) \subseteq A$  ... (i), and from 2., we set  $A \subseteq c_{qs}(A)$  ... (ii)  
Hence  $A \subseteq c_{qs}(A)$  by (i), (ii).  $\#$

**Theorem 3.3.29** Let  $\left( \bigoplus_{i \in I} X_i, q_s \right)$  be a  $q_s$ -space, and  $A, B \subseteq \bigoplus_{i \in I} X_i$ . Then

1.  $c_{qs}(A) \cup c_{qs}(B) = c_{qs}(A \cup B)$ ,
2.  $c_{qs}(A \cap B) \subseteq c_{qs}(A) \cup c_{qs}(B)$ .

**Proof 1.** Since  $A \subseteq A \cup B$  and  $B \subseteq A \cup B$ , by Theorem 3.3.28 3., we have

$$(A) \cup c_{qs}(B) \subseteq c_{qs}(A \cup B) \quad \dots (i).$$

Since  $c_{qs}(A) \cup c_{qs}(B)$  is a  $q_s$ -closed set containing  $A \cup B$  and also  $c_{qs}(A \cup B)$  is a  $q_s$ -closed set containing  $A \cup B$ .

$$\text{Hence by Theorem 3.3.28 5. } c_{qs}(A \cup B) \subseteq c_{qs}(A) \cup c_{qs}(B) \quad \dots (ii).$$

Therefore  $c_{qs}(A) \cup c_{qs}(B) \subseteq c_{qs}(A \cup B)$  by (i), (ii).  $\#$



2. Since  $A \cap B \subseteq A$  and  $A \cap B \subseteq B$ , by Theorem 3.3.28 2. , we have  
 $c_{qs}(A \cap B) \subseteq c_{qs}(A) \cup c_{qs}(B)$ . #.

The quality of Theorem 3.3.29 2. need not be true, see that following Example 3.3.30

**Example 3.3.30** Let  $I = \{1, 2\}$  ,  $X_1 = \{a, b\}$  ,  $X_2 = \{c, d\}$  ,  $q_1 = \{\{a\}, \{a, b\}\}$  ,  
 $q_2 = \{\{d\}, \{\}\}$  . We have  $\bigoplus_{i \in I} X_i = \bigcup_{i \in I} X_i = \{a, b, c, d\}$  ,  $q_s = \{\{a, d\}, \{a, b, d\}, \{a\}, \{a, b\}\}$  ,  
 $q_s - c \left( \bigoplus_{i \in I} X_i \right) = \{c, d\}, \{c\}, \{b, c, d\}, \{c, d\}$  .

Let  $A = \{a, b, c\}$  ,  $B = \{c\}$  , we have  $A \cap B = \{c\} \Rightarrow c_{qs}(A \cap B) = \{c\}$  ,  
 $c_{qs}(A) = \emptyset$  ,  $c_{qs}(B) = \{c\}$  .

We have  $c_{qs}(A) \cap c_{qs}(B) = \emptyset$  .

**Theorem 3.3.31** Let  $\left( \bigoplus_{i \in I} X_i, q_s \right)$  be a  $q_s$ -space, and  $A \subseteq \bigoplus_{i \in I} X_i$  . Then

$$1. \left[ i_{q_s}(A) \right]^c = c_{qs}(A)^c ,$$

$$2. c_{qs}(A)^c = \left[ i_{q_s}(A) \right]^c .$$

**Proof** 1. By definition of  $i_{q_s}(A)$  we have

$$i_{q_s}(A) = \bigcup \left\{ V : V \in q_s - o \left( \bigoplus_{i \in I} X_i \right) \text{ and } V \subseteq A \right\} .$$

$$\begin{aligned} \text{So, } \left[ i_{q_s}(A) \right]^c &= \left\{ \bigcup \left\{ V : V \in q_s - o \left( \bigoplus_{i \in I} X_i \right) \text{ and } V \subseteq A \right\} \right\}^c , \\ &= \bigcup \left\{ V : V \in q_s - o \left( \bigoplus_{i \in I} X_i \right) \text{ and } V \subseteq A \right\}^c , \\ &= \bigcup \left\{ V^c : V \in q_s - o \left( \bigoplus_{i \in I} X_i \right) \text{ and } V \subseteq A \right\} , \\ &= \bigcup \left\{ V^c : V^c \in q_s - c \left( \bigoplus_{i \in I} X_i \right) \text{ and } A^c \subseteq V^c \right\} , \\ &= \bigcup \left\{ F : F \in q_s - c \left( \bigoplus_{i \in I} X_i \right) \text{ and } A^c \subseteq F \right\} , \\ &= c_{qs}(A) . \end{aligned}$$

#.

2. By definition of  $c_{qs}(A)$  , we have

$$c_{qs}(A) = \bigcup \left\{ F \in q_s - c \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\} .$$

$$\begin{aligned} \text{So, } \left[ i_{q_s}(A) \right]^c &= \left[ \bigcup \left\{ F \in q_s - c \left( \bigoplus_{i \in I} X_i \right) : A \subseteq F \right\} \right]^c , \\ &= \bigcup \left\{ F^c \in q_s - c \left( \bigoplus_{i \in I} X_i \right) : F^c \subseteq A^c \right\} , \\ &= \bigcup \left\{ V \in q_s - c \left( \bigoplus_{i \in I} X_i \right) : V \subseteq A^c \right\} , \\ &= i_{q_s}(A^c) . \end{aligned}$$

#.

**Theorem 3.3.32** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space, and  $A \subseteq \bigoplus_{i \in I} X_i$ . Then

$$\bigcup_{i \in I} c_{qi}(A \cap X_i) \subseteq c_{qs}(A).$$

**Proof** For  $A \subseteq \bigoplus_{i \in I} X_i$ , since  $A \cap X_i \subseteq A, i \in I$ .

We have  $\bigcup_{i \in I} c_{qi}(A \cap X_i) \subseteq c_{qi}(A)$  By Note 3.3.26,

$$\text{we have } \bigcup_{i \in I} c_{qi}(A \cap X_i) \subseteq \bigcup_{i \in I} c_{qi}(A) \subseteq c_{qs}(A). \quad \#$$

**Example 3.3.33** Let  $I = \{1, 2\}$ ,  $X_1 = \{a, b\}$ ,  $X_2 = \{c, d\}$ ,  $q_1 = \{\{a\}, \{a, b\}\}$ ,  $q_2 = \{\{c\}\}$ ,  $q_s = \{\{a, c\}, \{a, b, c\}\}$ . Let  $A = \{a, b\}$ , we have  $\bigcup_{i \in I} c_{qi}(A \cap X_i) = \{a, b, d\} \subseteq c_{qs}(A) = \{a, b, c, d\}$ .

**Remark 3.3.34** From Theorem 3.3.32, if  $A$  is  $q_s$ -closed set in  $\bigoplus_{i \in I} X_i$ , we have  $c_{qs}(A) = \bigcup_{i \in I} (A \cap X_i)$ .

**Theorem 3.3.35** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space, and  $Y \subseteq \bigoplus_{i \in I} X_i$ . Then the collection  $q_{sY} = \{G \cap Y : G \in q_s\}$  is a  $q_s$ -space on  $Y$ .

**Proof** Let  $G \in q_s$ -o $\left(\bigoplus_{i \in I} X_i\right)$  be arbitrary and  $Y \subseteq \bigoplus_{i \in I} X_i$ . Since  $G \cap Y = G \cap X_i \subseteq X_i = Y$ , by Definition 3.1.4, we have  $G \cap X_i \subseteq q_i \forall i \in I$  that is  $G \cap Y = q_{sY}$ . Therefore  $q_{sY}$  be a quasi-generalized weak structure for  $Y$  generated by  $q_s$  #

**Definition 3.3.36** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space,  $Y \subseteq \bigoplus_{i \in I} X_i$  and the collection  $q_{sY}$  from Theorem 3.3.35 we called  $(Y, q_{sY})$  be a  $q_s$ -subspace of the space  $\left(\bigoplus_{i \in I} X_i, q_s\right)$ . The element of  $q_{sY}$  be called  $q_{sY}$ -open sets and complement of  $q_{sY}$ -open sets, it called  $q_{sY}$ -closed sets.

**Example 3.3.37** Let  $I = \{a, b, c\}$ ,  $X_a = \{1, 2\}$ ,  $X_b = \{3, 4\}$ ,  $X_c = \{5, 6\}$ ,  $q_a = \{\{1\}, \{1, 2\}\}$ ,  $q_b = \{\{3\}\}$ ,  $q_c = \{\{5\}, \{5, 6\}\}$ . We have  $\bigoplus_{i \in I} X_i = \{1, 2, 3, 4, 5, 6\}$  and  $q_s = \{\{1, 3, 6\}, \{1, 3, 5, 6\}, \{1, 2, 3, 6\}, \{1, 2, 3, 5, 6\}\}$ .

Let 1.  $Y = \{1, 3, 5\}$  we have

$$q_{sY} = \{\{1, 3\}, \{1, 3, 5\}\} \text{ and}$$

$$q_{sY}\text{-closed sets are: } \emptyset, \{5\}.$$

2.  $Z = \{1, 3, 6\}$  we have  $q_{sZ} = \{\{1, 3, 6\}\}$  and  $q_{sZ}$ -closed set is  $\emptyset$ .

**Note 3.3.38** Let  $A \subseteq Y \subseteq \bigoplus_{i \in I} X_i$ ,



1. If  $A$  is  $q_{SY}$ -open in  $Y$  and  $Y$  is  $q_s$ -open in  $\bigoplus_{i \in I} X_i$ , then  $A$  is  $q_s$ -open in  $\bigoplus_{i \in I} X_i$ .
2. The  $q_{SY}$  did not necessary a subset of  $q_s$ .

**Corollary 3.3.39** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space and  $X \subseteq \bigoplus_{i \in I} X_i$  then

$\{(X_i, q_i) : i \in I\} = \{(X_i, q_{sX_i}) : i \in I\}$  be a collection of  $q_s$ -subspace of  $\left(\bigoplus_{i \in I} X_i, q_s\right)$ .

**Proof** Obvious by Theorem 3.3.35.

**Theorem 3.3.40** Let  $(X_i, q_i)$  be  $q_s$ -subspace of  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  and  $A \subseteq X_i$  is  $q_s$ -closed set if and only if there exist a set  $F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right)$  such that  $A = F \cap X_i$ .

**Proof**  $A$  is  $q_s$ -closed set in  $X_i$

$$\Leftrightarrow X_i - A \text{ is } q_s\text{-open set in } X_i,$$

$$\Leftrightarrow X_i - A = G \cap X_i \exists G \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right),$$

$$\Leftrightarrow A = X_i - (G \cap X_i) = (X_i - G) \cup (X_i - X_i),$$

$$\Leftrightarrow A = X_i - G,$$

$$\Leftrightarrow A = X_i \cap G^C \text{ where } G^C \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right),$$

$$\Leftrightarrow A = X_i \cap F, \text{ where } F = G^C.$$

#.

**Theorem 3.3.41** Let  $(X_i, q_i)$  be a  $q_s$ -subspace of  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  and  $A \subseteq X_i$

then  $c_{qsX_i}(A) = c_{qs}(A) \cap X_i$ .

**Proof** By Definition 3.3.25,

$$\begin{aligned} c_{qsX_i}(A) &= \cap \left\{ F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right) : A \subseteq F \right\}, \\ &= \cap \left\{ F \cap X_i : F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right) \text{ and } A \subseteq F \cap X_i \right\}, \\ &= \cap \left\{ F \cap X_i : F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right) \text{ and } A \subseteq F \right\}, \\ &= \cap \left\{ F \cap X_i : F \in q_s\text{-}C\left(\bigoplus_{i \in I} X_i\right) \text{ and } A \subseteq F \right\} \cap X_i, \\ &= c_{qs}(A) \cap X_i. \end{aligned}$$

#.

### 3.4 $q_s$ -kernel

**Definition 3.4.42** Let  $\left(\bigoplus_{i \in I} X_i, q_s\right)$  be a  $q_s$ -space and  $A \subseteq \bigoplus_{i \in I} X_i$ . The  $q_s$ -kernel of  $A$  denoted as  $K_{qs}$ , is defined by  $K_{qs}(A) = \cap \left\{ V : A \subseteq V \text{ and } V \in q_s\text{-}O\left(\bigoplus_{i \in I} X_i\right) \right\}$ .

**Example 3.4.43** Let  $I = \{a, b\}$ ,  $X_a = \{1, 3\}$ ,  $X_b = \{2, 4\}$ ,  $q_a = \{\{3\}, \{1, 3\}\}$ ,  $q_b = \{\{2\}, \{2, 4\}\}$ . We have  $\bigoplus_{i \in I} X_i = \{1, 2, 3, 4\}$ ,  $q_s = \{\{2, 3\}, \{2, 3, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\}$ .

Then 1.  $K_{q_s}(\{2, 3\}) = \bigcap \{V : \{2, 3\} \subseteq V \text{ and } V \in q_s - \mathcal{O}(\bigoplus_{i \in I} X_i)\}$   
 $= \bigcap \{\{2, 3\}, \{2, 3, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\}$   
 $= \{2, 3\}$ .

2.  $K_{q_s}(\emptyset) = \{2, 3\}$ .

3.  $K_{q_s}(\{1, 2, 3, 4\}) = \{1, 2, 3, 4\}$ .

**Note 3.4.44** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space and  $A \subseteq \bigoplus_{i \in I} X_i$ . Then

1.  $K_{q_s}(A) \in q_s - \mathcal{O}(\bigoplus_{i \in I} X_i)$ ,

2.  $K_{q_s}(\bigoplus_{i \in I} X_i) = \bigoplus_{i \in I} X_i$ .

**Theorem 3.4.45** Let  $(\bigoplus_{i \in I} X_i, q_s)$  be a  $q_s$ -space and  $A, B$  are nonempty subset of  $\bigoplus_{i \in I} X_i$ . Then

1.  $A \subseteq K_{q_s}(A)$ ,

2.  $A \in q_s - \mathcal{O}(\bigoplus_{i \in I} X_i)$  then  $A = K_{q_s}(A)$ ,

3.  $K_{q_s}(K_{q_s}(A)) = K_{q_s}(A)$ ,

4.  $A \subseteq B$  then  $K_{q_s}(A) \subseteq K_{q_s}(B)$ ,

5.  $K_{q_s}(A \cup B) = K_{q_s}(A) \cup K_{q_s}(B)$ ,

6.  $K_{q_s}(A \cap B) = K_{q_s}(A) \cap K_{q_s}(B)$ .

**Proof** 1. Obviously by Definition 3.4.42 and we can say,  $K_{q_s}(A)$  is the smallest  $q_s$ -open set containing  $A$ . #.

2. Suppose  $A$  is  $q_s$ -open set. We want to prove  $A = K_{q_s}(A)$ .

By 1., we have  $A \subseteq K_{q_s}(A)$ . ... (i).

We shall show that  $K_{q_s}(A) \subseteq A$ .

Let  $x \in K_{q_s}(A) \Rightarrow x \in \bigcap V, A \subseteq V, V \in q_s - \mathcal{O}(\bigoplus_{i \in I} X_i)$

Since  $A$  is  $q_s$ -open, we have  $x \in \bigcap V = A$ .

Hence  $K_{q_s}(A) \subseteq A$ . ... (ii).

From (i), (ii). we have  $A = K_{q_s}(A)$ .



Conversely, suppose  $A = K_{qs}(A)$ , we have  $A$  is  $q_s$ -open set, since  $K_{qs}(A)$  is  $q_s$ -open. #.

3. Since,  $K_{qs}(A)$  be  $q_s$ -open set by 2. We have  $K_{qs}(K_{qs}(A)) = K_{qs}(A)$ . #.

4. By 1. we have  $A \subseteq K_{qs}(A)$  and  $B \subseteq K_{qs}(B)$ .

Since,  $A \subseteq B \Rightarrow A \subseteq K_{qs}(B)$  and by 1.

We obtained  $K_{qs}(A) \subseteq K_{qs}(B)$ . #.

5. By Definition 3.4.42, we have

$$\begin{aligned} x \in K_{qs}(A \cup B) &\Leftrightarrow x \in \bigcap \left\{ V : A \cup B \subseteq V, V \in q_s - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right) \right\}. \\ &\Leftrightarrow x \in \bigcap \left\{ V : A \subseteq V, V \in q_s - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right) \right\} \vee x \in \bigcap \left\{ V : B \subseteq V, V \in q_s - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right) \right\}. \\ &\Leftrightarrow x \in \bigcap K_{qs}(A) \cup \bigcap K_{qs}(B). \\ \text{Hence } K_{qs}(A) &= K_{qs}(A) \cup K_{qs}(B). \end{aligned} \quad \#.$$

6. We can prove it by Definition 3.4.42 as same as 5.

**Theorem 3.4.46** Let  $\left( \bigoplus_{i \in I} X_i, q_s \right)$  be a  $q_s$ -space and  $A \subseteq \bigoplus_{i \in I} X_i$ . Then

$$\bigcup_{i \in I} K_{qi}(A \cap X_i) \subseteq K_{qs}(A).$$

**Proof** Since  $A \cap X_i \in q_i$  and  $A \cap X_i \subseteq A \forall i \in I$ , so that  $A \in q_s$

$$\begin{aligned} \text{Let } x \in K_{qs}(A) &\Rightarrow x \in \bigcap \left\{ V : A \subseteq V, V \in q_i - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right) \right\} \\ &\Rightarrow x \in \bigcap \left\{ V : A \cap X_i \subseteq V, V \in q_i - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right) \right\}, \\ &\Rightarrow x \in K_{qi}(A \cap X_i). \end{aligned}$$

Hence  $\bigcup_{i \in I} K_{qi}(A \cap X_i) \subseteq K_{qs}(A)$ . #.

**Example 3.4.47** By Example 3.3.30, let  $A = \{a, c\}$ , we have

$$\bigcup_{i \in I} K_{qi}(A \cap X_i) = \{a, c, d\} \subseteq K_{qs}(A) = \{a, b, c, d\}.$$

**Note 3.4.48** By Theorem 3.4.46, let  $A \in q_s - \mathcal{O} \left( \bigoplus_{i \in I} X_i \right)$  then  $\bigcup_{i \in I} K_{qi}(A \cap X_i) = K_{qs}(A)$  #.

## Conclusion

The purpose of this research was to introduce a generalization of quasi generalized weak structure namely quasi generalized weak sum space (briefly,  $q_s$ -space), and to study some fundamental of qualitative intrinsic properties in term of  $q_s$ -interior,  $q_s$ -closure and  $q_s$ -kernel. We found that these sets were working on  $q_s$ -space and gives some fundamental properties under basic operations on sets. Furthermore, it can be show that the  $q_s$ -closure



and  $q_s$ -kernel sets were containing the arbitrary union of intersection of  $q_i$ -closure and  $q_i$ -kernel for any set  $A$  and  $X_i$  on the space  $\left(\bigoplus_{i \in I} X_i, q_i\right)$ ,  $\forall i \in I$  respectively, but not for  $q_s$ -interior, it is contained in the arbitrary union of intersection of  $q_i$ -interior for any set  $A$  and  $X_i$  on the space  $\left(\bigoplus_{i \in I} X_i, q_i\right)$ ,  $\forall i \in I$ . In the future, we shall investigate  $q_s$ -derived,  $q_s$ -exterior and  $q_s$ -boundary sets, and study to fundamental properties of them.

### Acknowledgements

Express thanks to Assoc. Prof. Ardoon Jongrak from Department of Mathematics Phetchabun Rajabhat University and Asst. Prof. Dr. Gumpol Sritanratana from Department of Mathematics Mahasarakham Rajabhat University, for their kind comments which resulted in an improved presentation of this paper. Thanks to Department of Mathematics, Faculty of Education of Chaiyaphum Rajabhat University for equipment support.

### References

- Ávila, J., and Molina F. (2012). Generalized Weak Structures. *International Mathematical Forum*. 7(52) : 2589-2595.
- Császár, A. (2012). Generalized open sets. *Acta Mathematica. Hungarica*. 137(3): 224-229.
- Janrongkam, P. (2019). *Quasi generalized weak structure*. B. Sc. Mahasarakham: Rajabhat Mahasarakham University.
- Thongpan, J. (2019). *Bi-quasi generalize weak structures*. B. Sc. Mahasarakham: Rajabhat Mahasarakham University.
- Zvina. I. (2011). Introduction to generalized topological spaces. *Applied General Topology*. 12(1): 49-66.

กลวิธีการสอนเรียบเรียงประโยคภาษาไทยสำหรับผู้เรียนชาวอินโดนีเซีย  
โดยใช้ความรู้จากคำศัพท์เรื่องการเดินทาง  
Teaching strategies for making Thai sentences to Indonesian learners  
Using travel vocabulary

สุทธิยา มหาเจริญ<sup>1\*</sup> และจอมขวัญ สุทธินนท์<sup>2</sup>  
Suttiya Mahajaroen<sup>1\*</sup> and Jomkwan Sudhinont<sup>2</sup>

<sup>1,2</sup> คณะศิลปศาสตร์ มหาวิทยาลัยสงขลานครินทร์ จังหวัดสงขลา 90110

<sup>1,2</sup> Faculty of Liberal Arts, Prince of Songkla University,  
Songkhla 90110, Thailand

<sup>1\*</sup> Corresponding Author, E-mail: dah\_suttiya@hotmail.com

Received: 12 July 2020, Revised: 28 November 2020, Accepted: 2 December 2020

#### บทคัดย่อ

บทความวิชาการนี้มีวัตถุประสงค์เพื่อนำเสนอกลวิธีการสอนเรียบเรียงประโยคภาษาไทยสำหรับผู้เรียนชาวอินโดนีเซียโดยใช้ความรู้จากคำศัพท์ที่ได้เรียนมา จากการศึกษาพบว่าผู้เรียนในโครงการจำนวน 13 คน ได้เรียนการเรียบเรียงประโยคภาษาไทยโดยใช้ความรู้จากกิจกรรมการเรียนการสอนคำศัพท์เรื่องการเดินทางจัดการเรียนการสอนตามแนวคิดแบบเน้นประสบการณ์ผนวกกับวิธีการเรียนการสอนโดยใช้สมองเป็นฐานโดยเริ่มสอนเรียบเรียงประโยคภาษาไทยจากความรู้เดิมคือคำศัพท์เรื่องการเดินทาง และสอนโครงสร้างประโยคตัวอย่างโดยใช้คำศัพท์เรื่องการเดินทาง เพื่อเป็นแนวทางในการแต่งประโยคที่สามารถนำไปใช้ได้จริงในชีวิตประจำวัน มีการวัดและประเมินผลจากกิจกรรมเสริม คือกิจกรรมเติมถ้อยคำในบทสนทนาให้ถูกต้องผู้เรียนสัมผัสผลตามวัตถุประสงค์การสอน กล่าวคือผู้เรียนสามารถจดจำคำศัพท์ที่จะนำไปใช้เรียบเรียงประโยคได้และสามารถจดจำโครงสร้างประโยคเพื่อนำไปใช้จริงในชีวิตประจำวันได้

**คำสำคัญ :** กลวิธีการสอน, การเรียบเรียงประโยค, การสอนแบบเน้นประสบการณ์

#### Abstract

The purposes of this research article was to present teaching strategies for making Thai sentences to Indonesian learners using travel vocabulary they had learned. Thirteen students under the project PSU International Block Course in semester 1 of academic year 2018 were taught how to make Thai sentences with travel vocabulary they had learned. The teaching was based on the experiential learning (EL) and brain based learning (BBL) approaches and the teaching began from how to make sentences using travel vocabulary that the students had learned, followed by making sentences, which could be used in real