การศึกษาเซตอนุพัทธ์-q_s เซตภายนอก-q_s และเซตขอบ-q_s บนปริภูมิ-q_s
A Study of q_s-Derived, q_s-Exterior and q_s-Boundary Sets On q_s-Space

สัจจารักษ์ ลาดสูงเนิน
Sajjarak Ladsungnern

สาขาวิชาคณิตศาสตร์ คระครุศาสตร์ มหาวิทยาลัยราชภัฏชัยภูมิ จังหวัดชัยภูมิ 36000
Department Mathematics, Faculty of Education, Chaiyaphum Rajabhat University,
Chaiyaphum 36000, Thailand.
Corresponding Author, E-mail: Ladsungnern@gmail.com

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บทคัดย่อ

วัตถุประสงค์ของการวิจัยในครั้งนี้คือการศึกษาสมบัติพื้นฐานจากการดำเนินการของเซตอนุพัทธ์-q_s เซตภายนอก-q_s และเซตขอบ-q_s ซึ่งเป็นเซตใหม่ที่ได้นิยามขึ้นมาบนปริภูมิ-q_s จากการศึกษาพบว่าในแต่ละการดำเนินการบนเซตเหล่านี้ทำให้ได้สมบัติต่างๆเกิดขึ้นในแต่ละการดำเนินการบนเซตนั้นๆ ยิ่งกว่านั้นเราจะพบว่ามีเพียงเซตอนุพัทธ์-q_s เท่านั้นที่บรรจุยูเนียนใดๆของอินเตอร์เซกของเซตอนุพัทธ์-q_i สำหรับเซต A ใด ๆ กับเซต X_i บนปริภูมิ (X_i, q_i) ทุก $i \in I$ ซึ่งตรงข้ามกับเซตภายนอก-q_s และเซตขอบ-q_s กล่าวคือเซตทั้งสองนี้จะถูกบรรจุในยูเนียนใดๆของอินเตอร์เซกของเซตภายนอก-q_i (เซตขอบ-q_i ตามลำดับ) สำหรับเซต A ใด ๆ กับเซต X_i บนปริภูมิ (X_i, q_i) ทุก $i \in I$

คำสำคัญ ปริภูมิ-q_s เซตอนุพัทธ์-q_s เซตภายนอก-q_s เซตขอบ-q_s

Abstract

The purpose of this research was to study to fundamental properties of the operation of sets (q_s-derived, q_s- exterior and q_s-boundary) which were newly defied basing on q_s-space. The study found that each of the operation of those sets led to the activations of properties in each operation of set. These sets worked on q_s-space and indicated various fundamental properties under defining the definitions of the sets and basic operations concept of sets on q_s- space. Furthermore, only the q_s- derived was in any sets of the arbitrary union of intersection of q_i- derived, for any set A, and X_i on the space

$\left(X_i, q_i\right), \forall i \in I$ which was in contrary of q_s- boundary and q_s- exterior. They were contained of the union of intersection of q_i - boundary and q_i-exterior for any set A and X_i respectively, on the space $\left(X_i, q_i\right), \forall i \in I$

Keywords: q_s -space, q_s -derived, q_s -exterior and q_s -boundary.

1. Introduction

The extension of the space was very important way to generalized concept for researcher to study and investigate especially topological space. Zvina (Zvina, 2011) introduced the notion of generalized topological space (gt-space). Generalized topology of gt-space has the structure of frame and is closed under arbitrary union and finite intersection modulo subsets. The family of small subsets of a gt-space forms an ideal that was compatible with the generalized topology. Császár (Császá, 2011) was the first introduced the notion of generalized topological space, called the weak structure. After that, Ávila (Ávila,2012) extend the weak structure to generalized weak structure briefly, GWS. Recently, Janrongkam (Janrongkam, 2019) extend GWS to quasi generalized weak structure briefly. QGWS and Thongpan (Thongpan, 2019) extend the last space to bi-quasi generalized weak space. In 2021,Ladsungnern (Ladsungnern,2021) introduced the quasi generalized weak sum space(briefly, q_s -space), and studied to fundamental properties of q_s -interior, q_s -closure and q_s -kernel sets. In this paper, we are going to introduce a new class of set namely, q_s -derived, q_s -exterior and q_s -boundary. Also, study to theirs important properties on q_s -space.

2. Preliminaries

In this section, we are going to present some concepts of q_s -interior, q_s -closure also, some properties of them on q_s -space. All results were presented by Ladsungnern (2021).

Definition2.1 Let $\left\{\left(X_i, q_i\right): i \in I\right\}$ be any collection of pairwise disjoint quasi generalized weak spaces. Let $\bigoplus_{i\in I} X_i = \bigcup_{i\in I} X_i$ and q_s be a collection of subset of $\bigoplus_{i\in I} X_i$ defined as follow: $q_s = \left\{U : U \subseteq \bigoplus_{i\in I} X_i \text{ and } U \cap X_i \in q_i\right\}$

Then q_s is a quasi generalized weak structure for $\underset{i\in I}{\oplus} X_i$ and it called, the quasi generalized weak sum structure for $\underset{i\in I}{\oplus} X_i$ Then pair $\left(\underset{i\in I}{\oplus} X_i, q_i\right)$ is called the quasi generalized weak sum space of the space $\{(X_i, q_i) : i \in I\}$ and its shortly called, q_s -space.Each element of q_s is said to be q_s -open and the complement to q_s -open is called q_s -closed sets. Throughout this paper, there are the symbols for use as follow:

1. $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ denoted by a q_s- space of the space $\{(X_i, q_s) : i \in I\}$
2. q_s-$O\left(\underset{i\in I}{\oplus} X_i\right)$ denoted by the collection of all q_s- open sets on $\underset{i\in I}{\oplus} X_i$.
3. q_s-$C\left(\underset{i\in I}{\oplus} X_i\right)$ denoted by the collection of all q_s- closed sets on $\underset{i\in I}{\oplus} X_i$.

Theorem 2.2 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and $A \subseteq \underset{i\in I}{\oplus} X_i$ be a q_s -open set iff $A = \underset{i\in I}{\cup}\left(A \cap X_i\right)$

Corollary 2.3 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and $\forall\ G_i \in q_i$. Then $A \subseteq \underset{i\in I}{\oplus} X_i$ is q_s -open set iff $A = \underset{i\in I}{\cup}\left(A \cap G_i\right)$

Theorem 2.4 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and A,B are q_s -open sets. Then $A \cup B$ and $A \cap B$ are q_s -open sets

Theorem 2.5 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and A,B are q_s -closed sets. Then

1. $A \cup B$ is q_s -closed set
2. $A \cap B$ is q_s -closed set.

Definition 2.6 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space and let $x \in \underset{i\in I}{\oplus} X_i$. A subset N of $\underset{i\in I}{\oplus} X_i$ is said to be a q_s -neighborhood, written as nbd of x if there exists a q_s -open set G such that $x \in G \subseteq N$ .Similarly N is called a q_s -nbd of $A \subseteq \underset{i\in I}{\oplus} X_i$; if there exists a q_s -open set G such that $A \subseteq G \subseteq N$

Remark 2.7 In any q_s -space nbd of a point need not be q_s -open set. On the often hand every q_s -open set is q_s -nbd of each of its points.

Theorem 2.8 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and $x \in \underset{i\in I}{\oplus} X_i$ be arbitrary. If N_1, N_2 be q_s -nbd of x then $N_1 \cap N_2$ is also q_s -nbd of x .

Definition 2.9 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s -space, and $A \subset \underset{i\in I}{\oplus} X_i$. The set of all q_s -interior of A denoted by $i_{qs}(A)$ denoted by $i_{qs}(A) = \underset{i\in I}{\cup}\left\{U \in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right) : U \subseteq A\right\}$.

In other word, a point x∈A is said to be q_s -interior point of A iff there exist are q_s - open set U on $\underset{i\in I}{\oplus} X_i$ such that $x \in U \subseteq A$.

Example 2.10 Let $I = \{1,2,3\}$, $X_1 = \{a,b,c\}, X_2 = \{c,d\}, X_3 = \{c\}$, $q_1 = \{\{a\},\{a,b\}\}, q_2 = \{\{d\},\{c,d\}\}, q_3 = \{\{e\}\}$. We have $\underset{i\in I}{\oplus} X_i = \{a,b,c,d,e\}$ and $q_S = \{\{a,d,e\},\{a,c,d,e\},\{a,b,d,e\},\{a,b,c,d,e\}\}$. The q_S - $O\left(\underset{i\in I}{\oplus} X_i\right)$ are: $\{a,d,e\},\{a,c,d,e\},\{a,b,d,e\},\{a,b,c,d,e\}$. The q_S - $O\left(\underset{i\in I}{\oplus} X_i\right)$ are: $\{c,b\},\{b\},\{c\},\varnothing$ Let $A = \{a,b\}$, then $= \underset{i\in I}{\cup}\{\ \} = \varnothing$.

Theorem 2.11 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - space, and $A, B \subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $i_{qs}(\varnothing) = \varnothing$. 2. $i_{qs}\left(\underset{i\in I}{\oplus} X_i\right) \subseteq \underset{i\in I}{\oplus} X_i$

3. $i_{qs}(A) \subseteq A$ 4. If $A \subseteq B$ then $i_{qs}(A) \subseteq i_{qs}(B)$

5. $i_{qs}(A)$ is a q_s -open set . 6. $i_{qs}(A)$ is the largest q_s -open set contained in A.

Theorem 2.12 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - space, and $A \subseteq \left(\underset{i\in I}{\oplus} X_i\right)$. Then

1. $A \in q_S$ - $O\left(\underset{i\in I}{\oplus} X_i\right)$ if and only if $A = i_{qs}(A)$

2. $i_{qs}\left(i_{qs}(A)\right) = i_{qs}(A)$.

Theorem 2.13 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s - space, $A \subseteq \left(\underset{i\in I}{\oplus} X_i\right)$. Then $i_{qs}(A) \subseteq \underset{i\in I}{\cup} i_{qi}(A \cap X_i)$.

Theorem2.14 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s - space, and $A, B \subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $i_{qs}(A) \cup i_{qs}(B) \subseteq i_{qs}(A \cup B)$

2. $i_{qs}(A \cap B) = i_{qs}(A) \cap i_{qs}(B)$.

Definition 2.15 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - space, and $A \subseteq \underset{i\in I}{\oplus} X_i$. The q_s - closure of A, denoted by $c_{qs}(A)$, is denote by $c_{qs}(A) = \cap\left\{F \in q_s - o\left(\underset{i\in I}{\oplus} X_i\right) : A \subseteq F\right\}$

Theorem 2.16 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - space, and $A, B \subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $c_{qs}\left(\underset{i\in I}{\oplus} X_i\right)=\underset{i\in I}{\oplus} X_i$, 2. $A\subseteq c_{qs}(A)$,
3. If $A\subseteq B$ then $c_{qs}(A)\subseteq c_{qs}(B)$, 4. $c_{qs}(A)$ is a q_s -closed set ,
5. $c_{qs}(A)$ is the smallest q_s - closed set containing A, and
6. $A\in q_s$ - $C\left(\underset{i\in I}{\oplus} X_i\right)$ iff $c_{qs}(A)=A$.

Theorem 2.17 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and $A,B\subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $c_{qs}(A)\cup c_{qs}(B)=c_{qs}(A\cup B)$,
2. $c_{qs}(A\cap B)\subseteq c_{qs}(A)\cap c_{qs}(B)$.

Theorem 2.18 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s - space, and $A\subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $\left[i_{qs}(A)\right]^c=c_{qs}\left(A^c\right)$
2. $i_{qs}\left(A^c\right)=\left[c_{qs}(A)\right]^c$

Theorem 2.19 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space, and $A\subseteq \underset{i\in I}{\oplus} X_i$. Then

$$c_{qs}(A)\supseteq \underset{i\in I}{\cup}\ c_{qi}\left(A\cap X_i\right).$$

3. Main Results

In this section, we shall be present the new sets and study the important properties of them on q_s -space. These are following as:

3.1 q_s -derived set

Definition 3.1.1 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s -space and A$\subseteq \underset{i\in I}{\oplus} X_i$. The q_s -derived set of A denoted by $d_{qs}(A)$, is defined by

$d_{qs}(A)=\{x\in \underset{i\in I}{\oplus} X_i : U\cap\left(A-\{x\}\right)\neq\varnothing\ \ \forall U\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right)$ and $x\in U\}$ Each member of $d_{qs}(A)$ is called limit points of A.

Remark 3.1.2 From definition 3.1.1

$$x\notin d_{qs}(A)\Rightarrow \forall U\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right) \text{ with } x\in U,\ U\cap\left(A-\{x\}\right)=\varnothing$$

or $\Rightarrow \forall\ U\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right)$ with $x\in U$, $U\cap A=\varnothing$ or $\{x\}$

Example 3.1.3 Let $I=\{1,2,3\}, X_1=\{a,b\}, X_2=\{e\}, X_3\{d,e\}, q_1=\{\{a\},\{a,b\}\},$

$q_2=\{\{e\}\}, q_3=\{\{e\},\{d,e\}\} I=\{\ \}$ We have $\underset{i=1,2,3}{\oplus} X_i=\{a,b,c,d,e\}$ and

$q_s=\{\{a,c,e\},\{a,c,d,e\},\{a,b,c,e\},\{a,b,c,d,e\}\}$

Let $A=\{c,d\}$.Then $d_{qs}(A)=\{d,e,a,b\}$ we have $c\notin d_{qs}(A)$, since

$\{a,c,e\}\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right)$ such that $\{a,c,e\}\cap\{c,d\}-\{c\}=\varnothing$.

Theorem 3.1.4 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s - space and A, B be non-empty subset of $\underset{i\in I}{\oplus} X_i$. Then 1. $d_{qs}(\varnothing)=\varnothing$ 2. $x\in d_{qs}(A)\Rightarrow x\in d_{qs}(A\text{-}\{x\})$

3. $A\subseteq B\Rightarrow d_{qs}(A)\subseteq d_{qs}(B)$ 4. $d_{qs}(A\cup B)=d_{qs}(A)\cup d_{qs}(B)$

5. $d_{qs}(A\cap B)=d_{qs}(A)\cap d_{qs}(B)$.

Proof 1. Let $A=\varnothing$, by remark 3.1.2 2., we have $d_{qs}(\varnothing)=\varnothing$. #.

2. Let $x\in d_{qs}(A)$ be arbitrary. Then $U\cap(A\text{-}\{x\})\neq\varnothing$ ... (*)

This is true $\forall\ U\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right)$ with $x\in U$.

Now $U\cap((A\text{-}\{x\}\text{-}\{x\})=U\cap\left(A\cap\{x\}^c\cap\{x\}^c\right))$

$=U\cap\left(A\cap\{x\}^c\right)$

$=U\cap(A\text{-}\{x\})\neq\varnothing$ by (*)

Thus $U\cap[(A\text{-}\{x\})\text{-}\{x\}]\neq\varnothing$ This $\Rightarrow x\in d_{qs}(A\text{-}\{x\})$. #.

3. Let $x\in d_{qs}(A)$ Then $U\cap(A\text{-}\{x\})\neq\varnothing$

$\forall\ U\in q_s\text{-}O\left(\underset{i\in I}{\oplus} X_i\right), x\in U$ ... (*)

$A\subseteq B\Rightarrow(A\text{-}\{x\})\cap\ U\subseteq(B\text{-}\{x\})\cap U$

$\Rightarrow(B\text{-}\{x\})\cap\ U\neq\varnothing$

$\Rightarrow x\in d_{qs}(B)$ Hence $d_{qs}(A)\subseteq d_{qs}(B)$. #.

4. Since $A\subseteq A\cup B$, $B\subseteq A\cup B$, by 3.. We have

$d_{qs}(B)\subseteq d_{qs}(A\cup B)$ $d_{qs}(A)\subseteq d_{qs}(A\cup B)$,

From which we get $d_{qs}(A)\cup d_{qs}(B)\subseteq d_{qs}(A\cup B)$ ... (1)

Remain to prove that $d_{qs}(A\cup B)\subseteq d_{qs}(A)\cup d_{qs}(B)$

That is, any $x\in d_{qs}(A\cup B)\Rightarrow x\in d_{qs}(A)\cup d_{qs}(B)$ ...(*)

Suppose $x\notin\left[d_{qs}(A)\cup d_{qs}(B)\right]$

$\Rightarrow\ \sim\left(x\in d_{qs}(A)\cup d_{qs}(B)\right)$

$\Rightarrow\ \sim\left(x\in\left[U\cap\left(A\text{-}\{x\}\right)\neq\varnothing\right]\cup\left[U\cap\left(A\text{-}\{x\}\right)\neq\varnothing\right]\right)$

$\Rightarrow\ \sim\left(x\in\left[U\cap\left(A\cup B\right)\text{-}\{x\}\neq\varnothing\right]\right)$

$\Rightarrow\ \sim\left(x\in d_{qs}\left(A\cup B\right)\right)$

$\Rightarrow\ x\notin d_{qs}\left(A\cup B\right).$

This statement equivalence (*), we have $d_{qs}\left(A\cup B\right)\subseteq d_{qs}\left(A\right)\cup d_{qs}\left(B\right)$ ... (2)

From (1). and (2)., we have $d_{qs}\left(A\cup B\right)=d_{qs}\left(A\right)\cup d_{qs}\left(B\right)$. # 5. Since, $A\cap B\subseteq A$ and $A\cap B\subseteq B$

$\Rightarrow d_{qs}\left(A\cup B\right)\subseteq d_{qs}\left(A\right)$ and $d_{qs}\left(A\cup B\right)\subseteq d_{qs}\left(B\right)$

$\Rightarrow d_{qs}\left(A\cup B\right)\subseteq d_{qs}\left(A\right)\cup d_{qs}\left(B\right)$.#.

Theorem 3.1.5 Let $\left(\underset{i\in I}{\oplus}X_i,q_s\right)$ be a q_s - space and $A\subseteq\underset{i\in I}{\oplus}X_i$ is closed iff $d_{qs}\left(A\right)\subseteq A$.

Proof Let $A\subseteq\underset{i\in I}{\oplus}X_i$ be closed. To prove that $d_{qs}\left(A\right)\subseteq A$ By theorem 2.16 we have $A=c_{qs}\left(A\right)$. we shall show that $d_{qs}\left(A\right)\subseteq c_{qs}\left(A\right)$... (*)

Let $x\in d_{qs}\left(A\right)\Rightarrow U\cap\left(A\text{-}\{x\}\right)\neq\varnothing, x\in U\in q_s\text{-}O\left(\underset{i\in I}{\oplus}X_i\right)$.

Since $A\text{-}\{x\}\subseteq A$, we have $U\cap A\neq\varnothing$, and then $x\in c_{qs}\left(A\right)$ Hence $d_{qs}\left(A\right)\subseteq A$, by (*).

Conversely, suppose that such that $d_{qs}\left(A\right)\subseteq A$. To prove that A is closed. Let $x\notin A\Rightarrow\ x\notin d_{qs}\left(A\right)$

$\Rightarrow\exists\ U\in q_s\text{-}O\left(\underset{i\in I}{\oplus}X_i\right)$ with $x\in U$ such that $U\cap\left(A\text{-}\{x\}\right)=\varnothing$

$\Rightarrow U\cap A=\varnothing\because A\text{-}\{x\}\subseteq A, x\notin A$

$\Rightarrow U\subseteq A^c$ .We have, any $x\in A^c\Rightarrow\exists\ U\in q_s\text{-}O\left(\underset{i\in I}{\oplus}X_i\right)$ with $x\in U$ such that $U\subseteq A^c$

Hence, $x\in i_{qs}\left(A^c\right)$. Since $x\in A^c$ is arbitrary, showing there by A^c is q_s -open set, that is a is q_s - closed set. #.

Theorem 3.1.6 Let $\left(\underset{i\in I}{\oplus}X_i,q_s\right)$ be a q_s - space and $d_{qs}\left(A\right)$ be a q_s - derived set of A. Then $d_{qs}\left(A\right)$ is a q_s - open set.

Proof By theorem 3.1.5, we have $d_{qs}\left(A\right)$ is a closed set iff $d_{qs}\left(d_{qs}\left(A\right)\right)\subseteq d_{qs}\left(A\right)$. We shall show it is true. Let $x\in d_{qs}\left(d_{qs}\left(A\right)\right)\Rightarrow x$ is a limit point of $d_{qs}\left(A\right)$ so that

$U \cap \left(d_{qs}(A)\text{-}\{x\} \neq \emptyset \;\forall\; U \in q_s\text{-}O\left(\bigoplus_{i\in I} X_i\right)\right)$ with $x \in U$ .This gives,

$U \cap \left(A\text{-}\{x\}\right) \neq \emptyset \Rightarrow x \in d_{qs}(A)$. #.

Theorem 3.1.7 Let $\left(\bigoplus_{i\in I} X_i, q_s\right)$ be a q_s - space, and $A \subseteq \bigoplus_{i\in I} X_i$, then $A \cup d_{qs}(A)$ is q_s - closed set.

Proof We shall prove that $\left[A \cup d_{qs}(A)\right]^c$ is q_s - open set.

Let $x \in \left[A \cup d_{qs}(A)\right]^c \Rightarrow \;\sim\left(x \in A \cup d_{qs}(A)\right)$

$\Rightarrow \;\sim\left(x \in A \vee x \in d_{qs}(A)\right)$

$\Rightarrow x \notin A \wedge x \notin d_{qs}(A)$

$x \in d_{qs}(A) \Rightarrow \exists\, U \in q_s\text{-}O\left(\bigoplus_{i\in I} X_i\right)$ with $x \in U$ such that $U \cap \left(A\text{-}\{x\}\right) \neq \emptyset$

$\Rightarrow U \cap A = \emptyset \because x \notin A$ ... (1)

For this U , we also claim $U \cap d_{qs}(A) \neq \emptyset$ ... (2)

Let $y \in U$ be arbitrary, now U is q_s - open set containing y such that $U \cap A = \emptyset$. Showing that $y \notin d_{qs}(A)$. Thus any $y \in U \Rightarrow y \notin d_{qs}(A)$, this proves $U \cap d_{qs}(A) = \emptyset$.

So we have $U \cap A = \emptyset$ and $U \cap d_{qs}(A) = \emptyset$.

Now, $U \cap \left[A \cup d_{qs}(A)\right] = (U \cap A) \cup \left[U \cap d_{qs}(A)\right] = \emptyset$.

This, give $\Rightarrow U \subseteq \left[A \cup d_{qs}(A)\right]^c$.

Hence, any $x \in \left[A \cup d_{qs}(A)\right]^c \Rightarrow \exists\, U \; q_s\text{-}O\left(\bigoplus_{i\in I} X_i\right)$ with $x \in U$ such that $\Rightarrow U \subseteq \left[A \cup d_{qs}(A)\right]^c$ .This proves that $x \in i_{qs}\left[A \cup d_{qs}(A)\right]^c$.

Therefore, $\left[A \cup d_{qs}(A)\right]^c$ is q_s - open set.. We obtain, $A \cup d_{qs}(A)$ is q_s - closed set #.

Corollary 3.1.8 Let $\left(\bigoplus_{i\in I} X_i, q_s\right)$ be a q_s - space, and $A \subseteq \bigoplus_{i\in I} X_i$ .Then $A \cup d_{qs}(A)$ is smallest q_s - closed set containing A .

Proof By theorem 3.1.5, 3.1.6 and $A \subseteq A \cup d_{qs}(A)$. #.

Theorem 3.1.9 Let $\left(\bigoplus_{i\in I} X_i, q_s\right)$ be a q_s - space, and $A \subseteq \bigoplus_{i\in I} X_i$ .Then $c_{qs}(A) = A \cup d_{qs}(A)$

Proof By theorem 3.1.5 and corollary 3.1.8. #.

Theorem 3.1.10 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - space and $A \subseteq \underset{i\in I}{\oplus} X_i$, then $d_{qs}(A) \supseteq \underset{i\in I}{\cup} d_{qi}(A \cap X_i)$

Proof Since, $A \cap X_i \subseteq A$, by theorem 3.1.4.We have, $c_{qs}(A \cap X_i) \subseteq d_{qs}(A)$

So, $\underset{i\in I}{\cup} d_{qi}(A \cap X_i) \subseteq d_{qs}(A)$.#.

Corollary3.1.11 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$be a q_s - space and $A \in q_S$- $C\left(\underset{i\in I}{\oplus} X_i\right)$then $d_{qs}(A) = \underset{i\in I}{\cup} d_{qi}(A \cap X_i)$

Proof By theorem 3.1.9 we have $d_{qs}(A) \subseteq c_{qs}(A) = A \subseteq \underset{i\in I}{\cup} c_{qi}(A \cap X_i)$ Andbytheorem3.1.9again, we have $d_{qs}(A) \subseteq \underset{i\in I}{\cup} d_{qi}(A \cap X_i)$By theorem 3.1.10 we have $d_{qs}(A) = \underset{i\in I}{\cup} d_{qi}(A \cap X_i)$. #.

3.2 q_S -exterior

Definition 3.2.12 Let $\left(\underset{i\in I}{\oplus} X_i, q_S\right)$be a q_s - space, and $A \subseteq \underset{i\in I}{\oplus} X_i$. A point $x \in \underset{i\in I}{\oplus} X_i$ is said to be q_s - exterior point of A if it is q_s - interior point of A^C. The set of all q_s - exterior point of A denoted by $e_{qs}(A)$. That is $e_{qs}(A) = \left\{x \in \underset{i\in I}{\oplus} X_i : x \in i_{qs}(A^c)\right\}$

Note3.2.13 By definition 3.2.12 we can say $x \in \underset{i\in I}{\oplus} X_i$ be q_s -exterior point of A if there exists q_s - open set G such that $x \in G \subseteq A^c$ or equivalently $x \in G$ and $G \cap A = \varnothing$

Example3.2.14 By example 3.1.3Let $A = \{c,d\}$, then $A^c = \{a,b,e\}$. We have $i_{qs}(A) = \varnothing = e_{qs}(A)$. If $B = \{b\}$ then. $e_{qs}\{b\} = i_{qs}\{a,c,d,e\} = \{a,c,d,e\}$.

Remark 3.2.15 Since $e_{qs}(A)$ is the $i_{qs}(A^c)$, it follow from theorem 2.11that $e_{qs}(A)$ is q_s -open and is the largest q_s - open set contained in A^c.

Theorem3.2.16 Let$\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s -space and let $A \subseteq \underset{i\in I}{\oplus} X_i$.Them $e_{qs}(A) = U\left\{G \in q_s - O\left(\underset{i\in I}{\oplus} X_i\right) : G \subseteq A^c\right\}$

Proof Since $i_{qs}(A) = U\left\{G \in q_s - O\left(\underset{i\in I}{\oplus} X_i\right) : G \subseteq A^c\right\}$. By definition3.2.12, $e_{qs}(A) = i_{qs}(A^c)$. So we have, $i_{qs}(A^c) = U\left\{G \in q_s - O\left(\underset{i\in I}{\oplus} X_i\right) : G \subseteq A^c\right\}$. #.

Theorem 3.2.17 Let$\left(\underset{i\in I}{\oplus} X_i, q_S\right)$ be a q_s - spaceand let A,B be subsets of $\underset{i\in I}{\oplus} X_i$. Then

1. $e_{qs}\left(\underset{i\in I}{\oplus} X_i\right)=\varnothing, e_{qs}(\varnothing)=\underset{i\in I}{\oplus} X_i$, 2. $e_{qs}(A)\subseteq A^c$

3. $e_{qs}(A)=e_{qs}(A)\left[\left(e_{qs}(A)\right)^c\right]$ 4. $A\subseteq B\Rightarrow e_{qs}(A)\subseteq e_{qs}(B)$

5. $i_{qs}(A)\subseteq e_{qs}\left(e_{qs}(A)\right)$ 6. $e_{qs}(A\cup B)=e_{qs}(A)\cap e_{qs}(B)$

7. $e_{qs}(A\cup B)\supseteq e_{qs}(A)\cup e_{qs}(B)$

Proof1. $e_{qs}\left(\underset{i\in I}{\oplus} X_i\right)=i_{qs}\left[\left(\underset{i\in I}{\oplus} X_i\right)^c\right]=i_{qs}(\varnothing)=\varnothing$

$$e_{qs}(\varnothing)=i_{qs}(\varnothing)^c$$

$$=i_{qs}\left(\underset{i\in I}{\oplus} X_i\right)=\underset{i\in I}{\oplus} X_i$$.#.

2. $e_{qs}(A)=i_{qs}\left(A^c\right)\subseteq A^c$. #.

3. By 2. we have $A\subseteq\left[e_{qs}(A)\right]^c$ ... (i) .#.

We shall show that $\left[e_{qs}(A)\right]^c\subseteq A$. Let $x\in\left[e_{qs}(A)\right]^c\Rightarrow x\in A$

$\subseteq A$ ... (ii)

By (i), (ii) we have $A=,\left[e_{qs}(A)\right]^c$. That it is hold for 3 ..#.

4. $A\subseteq B\Rightarrow B^c\subseteq A^c$

$\Rightarrow i_{qs}\left(B^c\right)\subseteq i_{qs}\left(A^c\right)$

$\Rightarrow e_{qs}(B)\subseteq e_{qs}(A)$. #..

5. By 2., we have $e_{qs}(A)\subseteq A^c$. Then 4.gives $e_{qs}\left(A^c\right)\subseteq e_{qs}\left(e_{qs}\left(A^c\right)\right)$. But $i_{qs}(A)=e_{qs}\left(A^c\right)$. Hence $i_{qs}(A)\subseteq e_{qs}\left(e_{qs}(A)\right)$

6. $e_{qs}(A\cup B)\subseteq i_{qs}(A\cup B)=i_{qs}\left(A^c\cap B^c\right)=i_{qs}\left(A^c\right)\cap i_{qs}\left(B^c\right)$

$=e_{qs}(A)\cap e_{qs}(B)$. #.

7. $e_{qs}(A\cup B)=i_{qs}(A\cap B)^c=i_{qs}\left(A^c\cup B^c\right)$

$=i_{qs}\left(A^c\right)\cup i_{qs}\left(B^c\right)$

$=e_{qs}(A)\cup e_{qs}(B)$. #.

Theorem 3.2.18 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s-space, and $A\subseteq\underset{i\in I}{\oplus} X_i$. Then $e_{qs}(A)\subseteq\underset{i\in I}{\cup} e_{qi}\left(A\cap X_i\right)$

Proof By theorem 3.2.15 we have,

$x \in e_{qs}(A) \Rightarrow x \in G_i \ \exists\, i \in I, G_i \subseteq A^c \subseteq (A \cap X_i)^c$

$\Rightarrow x \in i_{qi}(A \cap X_i)^c \ \exists\, i \in I$

$\Rightarrow x \in \bigcup_{i \in I} e_{qi}(A \cap X_i)$ Hence $e_{qs}(A) \subseteq \bigcup_{i \in I} e_{qi}(A \cap X_i)$. #.

3.3 q_s - boundary sets

Definition 3.3.19 Let $\left(\bigoplus_{i \in I} X_i, q_s\right)$ be a q_s - space and $A \subseteq \bigoplus_{i \in I} X_i$. A point $x \in \bigoplus_{i \in I} X_i$ is said to be a q_s - boundary of A iff it is neither $i_{qs}(A)$ nor $e_{qs}(A)$. The set of all boundary points of A denoted by $b_{qs}(A)$

Note 3.3.20 By definition 3.3.19, we have

1. $x \in b_{qs}(A) \Leftrightarrow x \notin i_{qs}(A)$ and $x \notin e_{qs}(A) = i_{qs}(A^c)$

$\Leftrightarrow$ neither A nor A^c is a q_s -nbd of x

$\Leftrightarrow$ no q_s - nbd of x can be contained or A^c in A

$\Leftrightarrow$ every q_s - nbd of x intersecs both A and A^c

2. $b_{qs}(A) = [i_{qs}(A)]^c \cap [i_{qs}A^c]^c$

Example 3.3.21 From example 3.1.3Let $A = \{a,b,c,e\}$, we have $i_{qs}(A) = \{a,b,c,e\}$ and $e_{qs}(A) = \varnothing$.We obtained the set $\{d\}$ be a q_s -boundary point of A.

Theorem 3.3.22 Let $\left(\bigoplus_{i \in I} X_i, q_s\right)$be a q_s - space and $A \subseteq \bigoplus_{i \in I} X_i$. Then

1. $b_{qs}(A) = b_{qs}(A^c)$,

2. $b_{qs}(A)$ is a q_s -closed set.

Proof 1. by notice 3.3.20 1.. We have

$x \in b_{qs}(A) \Leftrightarrow$ every q_s - nbd of x intersects both A and A^c

$\Leftrightarrow$every q_s - nbd of x intersects $(A^c)^c$ and A^c. #.

2.. by notice 3.3.20 2.. We have $b_{qs}(A)$ is a union of q_s -closed sets .#

Theorem 3.3.23 Let $\left(\bigoplus_{i \in I} X_i, q_s\right)$ be a q_s - space and $A \subseteq \bigoplus_{i \in I} X_i$. Then

1. $i_{qs}(A), e_{qs}(A), b_{qs}(A)$ are disjoint sets.

2. $\bigoplus_{i \in I} X_i = i_{qs}(A) \cup e_{qs}(A) \cup b_{qs}(A)$

Proof 1. By definition $e_{qs}(A) = i_{qs}(A^c)$ .Also $i_{qs}(A) \subseteq A, i_{qs}(A^c) = A^c$.

Since $A \cap A^c = \varnothing$ it follows that $i_{qs}(A) \cap e_{qs}(A) = i_{qs}(A) \cap i_{qs}(A^c) = \varnothing$.

Again by the definition of $b_{qs}(A)$. We have

$$x \in b_{qs}(A) \Leftrightarrow x \notin i_{qs}(A) \text{ and } x \notin e_{qs}(A)$$
$$\Leftrightarrow x \notin \left[i_{qs}(A) \cup e_{qs}(A)\right]$$
$$\Leftrightarrow x \in \left[i_{qs}(A) \cup e_{qs}(A)\right]^c$$

Thus $b_{qs}(A) = \left[i_{qs}(A) \cup e_{qs}(A)\right]^c$ If follow that $b_{qs}(A) \cap i_{qs}(A) = \varnothing$ and $b_{qs}(A) \cap e_{qs}(A) = \varnothing$. #.

2. So, we have $\underset{i\in I}{\oplus} X_i = i_{qs}(A) \cap e_{qs}(A) \cup b_{qs}(A)$. #.

Theorem 3.3.24 Let $\left(\underset{i\in I}{\oplus} X_i, q_s\right)$ be a q_s - space and $A, B \subseteq \underset{i\in I}{\oplus} X_i$. Then

1. $b_{qs}(A) = c_{qs}(A)$, $i_{qs}(A) = c_{qs}(A) \cap c_{qs}(A^c)$, 2. $c_{qs}(A) = i_{qs}(A) \cup b_{qs}(A)$
3. $i_{qs}(A) = A - b_{qs}(A)$, 4. $b_{qs}(c_{qs}(A)) \subseteq b_{qs}(A)$
5. $b_{qs}(i_{qs}(A)) \subseteq b_{qs}(A)$ 6. $b_{qs}(A \cup B) \subseteq b_{qs}(A) \cup b_{qs}(B)$
7. $b_{qs}(A \cap B) \subseteq b_{qs}(A) \cup b_{qs}(B)$

Proof 1. By note 3.3.20 2., we have $b_{qs}(A) = \left[i_{qs}(A)\right]^c \cap \left[i_{qs} A^c\right]^c$ ... (i) .

Replacing A by A^c .By theorem 3.2.15, we have $\left[i_{qs}(A)\right]^c = c_{qs}(A^c)$ and $i_{qs}(A^c) = \left[c_{qs}(A)\right]^c$ ... (ii) .

Put (ii) in (i). we have

$$b_{qs}(A) = c_{qs}(A^c) \cap \left[c_{qs}(A)\right]^c = c_{qs}(A^c) \cap c_{qs}(A)$$
$$= c_{qs}(A) - \left[c_{qs}(A)\right]^c = c_{qs}(A) - i_{qs}(A). \quad \#.$$

2. We shall show that $b_{qs}(A) = e_{qs}(A) = i_{qs}(A) \cup b_{qs}(A)$ By 1.,

$$i_{qs}(A) \cup b_{qs}(A) = i_{qs}(A) \cup \left[c_{qs}(A^c) \cap c_{qs}(A)\right] = \left[i_{qs}(A) \cup c_{qs}(A)\right] \cap \left[i_{qs}(A) \cup c_{qs}(A)\right]$$
$$= c_{qs}(A) \cap \left[i_{qs}(A) \cup \left[i_{qs}(A)\right]^c\right] = c_{qs}(A). \quad \#.$$

3. $A - b_{qs}(A) = A - \left[c_{qs}(A) \cap c_{qs}(A^c)\right] = \left(A - c_{qs}(A)\right) \cup \left(A - c_{qs}(A^c)\right)$

$$= \varnothing \cup \left(A - \left[i_{qs}(A)\right]^c\right) = A - \left(i_{qs}(A)\right)^c$$
$$= A \cap i_{qs}(A), \text{ as } A - B = A \cap B^c$$
$$= i_{qs}(A), \text{ since } i_{qs}(A) \subseteq A \quad \#.$$

4. Since, $b_{qs}(A) = c_{qs}(A) \cap c_{qs}(A^c)$.

So, $b_{qs}\left(c_{qs}(A)\right)=c_{qs}\left(c_{qs}(A)\right)\cap c_{qs}\left(c_{qs}(A)\right)^c=c_{qs}(A)\cap c_{qs}\left(c_{qs}(A)\right)^c$

$=c_{qs}(A)\cap c_{qs}\left(i_{qs}(A)\right)^c$, since $c_{qs}(A)=\left[i_{qs}\left(A^c\right)\right]^c$

$=c_{qs}(A)\cap c_{qs}\left(A^c\right)=b_{qs}(A)$. #.

5. Since, $b_{qs}(A)=c_{qs}(A)\cap c_{qs}\left(A^c\right)$

So, $b_{qs}\left(i_{qs}(A)\right)=c_{qs}\left(i_{qs}(A)\right)\cap c_{qs}\left(i_{qs}\left(A^c\right)\right)\subseteq c_{qs}(A)\cap c_{qs}\left(A^c\right)=b_{qs}(A)$. #.

6. Aim $b_{qs}(A\cup B)\subseteq b_{qs}(A)\cup b_{qs}(B)$ Since $b_{qs}(A)=c_{qs}(A)\cap c_{qs}\left(A^c\right)$

$b_{qs}(B)=c_{qs}(B)\cap c_{qs}\left(B^c\right)$

Thus, $b_{qs}(A)\cup b_{qs}(B)=\left[c_{qs}(A)\cap c_{qs}\left(A^c\right)\right]\cup\left[c_{qs}(B)\cap c_{qs}\left(B^c\right)\right]$

$=\left[c_{qs}(A)\cap c_{qs}(B)\right]\cap\left[c_{qs}\left(A^c\right)\cap c_{qs}\left(B^c\right)\right]$

By theorem 2.17, we have $b_{qs}(A)\cup b_{qs}(B)=c_{qs}(A\cup B)\cap c_{qs}(A\cup B)^c$

$\supseteq b_{qs}(A\cup B)$ #..

7. We want to prove that $b_{qs}(A\cap B)\subseteq b_{qs}(A)\cup b_{qs}(B)$ Since $A\cap B\subseteq A\cup B$, by theorem 2.17 $c_{qs}(A\cap B)\subseteq c_{qs}(A)\cap c_{qs}(B)$ ... (i)

$c_{qs}(A\cup B)^c=c_{qs}\left(A^c\cup B^c\right)$

$=c_{qs}\left(A^c\right)\cap c_{qs}\left(B^c\right)$... (ii) From (i), (ii). We have

$c_{qs}(A\cap B)\cap c_{qs}(A\cup B)^c\subseteq\left[c_{qs}(A)\cap c_{qs}(B)\right]\cap\left[c_{qs}\left(A^c\right)\cup c_{qs}\left(B^c\right)\right]$

$b_{qs}(A\cap B)\subseteq\left[c_{qs}(A)\cap c_{qs}\left(A^c\right)\right]\cup\left[c_{qs}(B)\cap c_{qs}\left(B^c\right)\right]=b_{qs}(A)\cup b_{qs}(B)$. #.

Theorem 3.3.25 Let $\left(\underset{i\in I}{\oplus}X_i,q_s\right)$ be a q_s -space and let $A\subseteq\underset{i\in I}{\oplus}X_i$. Then

1. If A is q_s -open, then $b_{qs}(A)=c_{qs}(A)-A$

2. $b_{qs}(A)=\varnothing$ if and only if A is q_s -open as well as q_s - closed

3. A is q_s - open iff $A\cap b_{qs}(A)=\varnothing$, that is iff $b_{qs}(A)\subseteq c_{qs}(A)$

4. A is q_s - closed iff $b_{qs}(A)\subseteq A$

Proof 1. by theorem 3.3.21.. We have

$b_{qs}(A)=c_{qs}(A)-i_{qs}(A)$

$=c_{qs}(A)-A\quad\because i_{qs}(A)=A$. #.

2. We want to prove that

$b_{qs}(A)=\varnothing \Leftrightarrow A$ is q_s - open and q_s -closed.Let A is q_s - open and q_s - closed

$\Rightarrow i_{qs}(A)=A, c_{qs}(A)=A$

$\Rightarrow c_{qs}(A)\text{-}i_{qs}(A)=\varnothing \quad \Rightarrow b_{qs}(A)=\varnothing$

Conversely, let $b_{qs}(A)=\varnothing \Rightarrow c_{qs}(A)\text{-}i_{qs}(A)=\varnothing$

$\Rightarrow c_{qs}(A)\subseteq i_{qs}(A)$ ... (*)

But $i_{qs}(A)\subseteq A$, $\Rightarrow c_{qs}(A)\subseteq A$, but $A\subseteq c_{qs}(A)$

$\Rightarrow A=c_{qs}(A)$

$\Rightarrow A$ is q_s - closed By (*) $c_{qs}(A)=A\cup d_{qs}(A)\subseteq i_{qs}(A)$

$\Rightarrow A\subseteq A\cup d_{qs}(A)\subseteq i_{qs}(A)\subseteq A \Rightarrow i_{qs}(A)=A$

$\Rightarrow A$ is q_s -open .#.

3. Let A be q_s - open, there A^c is q_s - closed. Hence $c_{qs}(A^c)=A^c$.By theorem 3.3.22 1., we have

$$A\cap b_{qs}(A)=A\cap\left[c_{qs}(A)\cap c_{qs}(A^c)\right]=\left[A\cap c_{qs}(A)\right]\cap c_{qs}(A^c)$$
$$=A\cap c_{qs}(A^c)=A\cap A^c$$
$$=\varnothing$$

Conversely. Let $A\cap b_{qs}(A)=\varnothing$ By 1. we have

$$A\cap b_{qs}(A)=\varnothing \Rightarrow A\cap\left(c_{qs}(A)\cap c_{qs}(A^c)\right)=\varnothing \Rightarrow A\cap c_{qs}(A^c)=\varnothing$$
$$\Rightarrow A\subseteq\left[c_{qs}(A^c)\right]^c=\varnothing \Rightarrow A\subseteq i_{qs}(A)$$

But $i_{qs}(A)\subseteq A$, Hence $i_{qs}(A)=A$. If follows that A is q_s - open.

4. Let A be q_s -closed. Then $c_{qs}(A)=A$

Hence $b_{qs}(A)=c_{qs}(A)\cap c_{qs}(A^c)=A\cap c_{qs}(A^c)=A$

Conversely, let $b_{qs}(A)\subseteq A \Rightarrow A\cup b_{qs}(A)=A$ But $A\cup b_{qs}(A)=c_{qs}(A)$.It follow that $A=c_{qs}(A)$ Hence A is q_s - closed. #.

Theorem 3.3.26 Let $\left(\underset{i\in I}{\oplus}X_i, q_s\right)$ be a q_s -space and $A\subseteq \underset{i\in I}{\oplus}X_i$.Then $b_{qs}(A)\subseteq \underset{i\in I}{\cup} b_{qs}(A\cap X_i)$.

Proof By theorem 3.3.24, we have $b_{qs}(A)=c_{qs}(A)\cap c_{qs}(A^c)$.

By theorem 2.19, we have $b_{qs}(A)\subseteq \underset{i\in I}{\cup} c_{qs}(A\cap X_i)\cap \underset{i\in I}{\cup} c_{qs}(A\cap X_i)^c$

$\subseteq \underset{i\in I}{\cup}\left[c_{qs}(A\cap X_i)\cap c_{qs}(A\cap X_i)^c\right]\subseteq \underset{i\in I}{\cup} b_{qs}(A\cap X_i)$. #.

Conclusion

The purposive of this paper was to introduce the q_s - exterior, q_s -derived and q_s -boundary and study the important properties of them on q_s - space. We obtained various important properties related to q_s -closure , q_s -interior and others on q_s - space. Furthermore, the research found that the q_s -exterior and q_s -boundary are contained in the union of intersection of q_i - exterior and q_i - boundary of any set A and X_i for the space $\left(\bigoplus_{i\in I} X_i, q_s\right)$ but, contrary for q_s - derived set i.e. it is containing the union of intersection of q_i -derived of any set A and X_i for the space . $(X_i, q_i) : i \in I$ In the future, we shall study to q_s -continuity and also, q_s - homeomorphism on this space.

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