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Exchange Rate Intervention and Degree of Openness

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Abstract

Since most of existing literature analyses exchange rate intervention in a partial equilibrium framework, implying that the spillover effect of the intervention on other sectors is ignored, in this paper, however, the problem will be posed under general equilibrium setting in which the spillover effect of exchange rate intervention will be taken into account. Using degree of openness as a proxy for the spillover effect of exchange rate intervention, it is postulated that, with high degree of openness, the spillover effect of the intervention is beneficial since it will further help stabilizing the whole economy. Hence, with high degree of openness, the optimal exchange rate band to be narrower than the one with low degree of openness, signifying the central bank's bias toward more intervention rather than floating of exchange rate. With high degree of openness, large portion of GDP is accounted for by international trade transactions. Therefore, stabilizing international trade sector via exchange rate intervention will have positive spillover effect by helping to further stabilize the whole economy. Exchange rate intervention should be, therefore, beneficial for a country in which degree of openness is relatively high.

Keyword: Exchange Rate Intervention, Degree of Openness, General Equilibrium

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การแทรกแซงอัตราแลกเปลี่ยน และอัตราการเปิดประเทศ

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บทคัดย่อ

วัตถุประสงค์หลักของการแทรกแซงอัตราแลกเปลี่ยนก็เพื่อลดความผันผวนซึ่งจะส่งผลต่อการประกอบธุรกรรมทางเศรษฐกิจระหว่างประเทศ เนื่องจากวรรณกรรมส่วนใหญ่ที่เกี่ยวกับการแทรกแซงอัตราแลกเปลี่ยนมักจะวิเคราะห์ปัญหาภายใต้กรอบดุลยภาพเฉพาะส่วน (Partial Equilibrium) ซึ่งไม่ได้นำผลกระทบของการแทรกแซงอัตราแลกเปลี่ยนต่อระบบเศรษฐกิจโดยรวมมาพิจารณา ในบทความนี้ การวิเคราะห์จะทำภายใต้กรอบดุลยภาพทั่วไป (General Equilibrium) เพื่อวิเคราะห์ผลกระทบจากการแทรกแซงอัตราแลกเปลี่ยนต่อภาคเศรษฐกิจอื่น (Spillover Effects) โดยอัตราการเปิดประเทศ (Degree of Openness) จะถูกนำมาใช้เป็นตัวแทน (Proxy) ของผลกระทบดังกล่าว จากการศึกษาพบว่า ประเทศที่มีอัตราการเปิดประเทศที่สูงจะได้รับประโยชน์จากการแทรกแซงอัตราแลกเปลี่ยน ในทางตรงข้าม ประเทศที่มีอัตราการเปิดประเทศที่ต่ำ ประโยชน์จากการแทรกแซงอัตราแลกเปลี่ยนจะมีน้อยจะไม่สามารถไปชดเชยกับต้นทุนที่เกิดจากการเคลื่อนย้ายทรัพยากรจากภาคในประเทศ

คำสำคัญ: การแทรกแซงอัตราแลกเปลี่ยน อัตราการเปิดประเทศ แบบจำลองดุลยภาพทั่วไป

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1. Introduction

In a liberalized global financial market, it is normal to see large amount of capital flows across national boundaries on a daily or, perhaps, hourly basis. These international capital flows will cause both positive and negative effects on the host country economic system. On a positive side, the capital inflows can help improve the liquidity in the financial market for the host country. But on a negative side, these capital flows, both in and out, can create instability for the host country economic system.

The instability caused by capital flows may be in large part due to the fluctuations in exchange rate, which will in turn transmits negative effects to the real sector of the economy. The fluctuations in exchange rate, which can be viewed as an uncertainty inherent to a small- open economy, may be a legitimate reason for the central bank to intervene in order to stabilize or provide a certain kind of insurance to enhance the performance of the economy.

It is interesting to study how the central bank intervene exchange rate in an optimal manner so as to obtain second best efficiency. Particularly, when the central bank is engaged in multi-objective policies, the decision to intervene in exchange rate will be very difficult to implement.

The existing literature on exchange rate intervention mostly assumes that the purpose of central bank intervention is to minimize the variance of exchange rate around its target level determined exogenously by the central bank, with a certain structure of intervention cost being given exogenously. Most of the literature focuses on the "Target Zone" regime of exchange rate which is currently adopted as a benchmark exchange rate policy by the European Central Bank (ECB).

In this paper, the author attempts to incorporate the managed floating exchange rate regime into the baseline model. The model is applied to the case of Thai Baht/US dollar exchange rate. The finding under this setting will be useful for the policy maker to optimally intervene exchange rate in a variety of situations.

Specifically, the problem for the central bank is to find the critical levels (upper and lower boundaries of the band) such that whenever the exchange rate process reaches either side of the boundaries, the central bank must intervene immediately so that the optimal exchange rate band can be established and consequently the objective cost function can be minimized.

With managed floating regime being incorporated into the model, the objective of exchange rate intervention will also involve the stabilization of the domestic economy as well, instead of stabilizing international trade sector alone as in the case of the baseline model. In this paper, the degree of openness is introduced as a key parameter that plays an important role in determining the optimal level of exchange rate intervention. Particularly, the degree of openness will be used as a proxy to determine whether the exchange rate intervention is beneficial or harmful and hence what level of exchange rate intervention should be optimally exercised by the central bank.

The paper is organized as follow: Section 1 is an introduction. Section 2 is a literature review. Section 3 explores the facts about the degree of openness of various countries with associated exchange rate regime adopted for use in each country. Section 4 presents an overview of the exchange rate intervention model, using impulse control framework that is currently employed in the existing literature. This model will be used as a baseline model for further extensions. In Section 5, we will extend the model to include managed floating regime where the degree of openness is introduced as a parameter that influences the optimal level of exchange rate intervention. The results from optimization are compared with those of the baseline model. Section 6 summarizes the findings. The policy implications of the model are discussed.

2. Literature Reviews

The objectives for exchange rate intervention under managed floating regime can be summarized as follow: (see Bank of Thailand, 2004)

1) To reduce exchange rate volatility
2) To make correction when exchange deviates significantly from its fundamentals
3) To adjust exchange rate to avoid speculative attack
4) To respond to adverse shocks
5) To accumulate and/or maintain proper level of foreign exchange reserves

Analytically, the objective of exchange rate intervention by the central bank is to minimize the total cost comprising the cost arising from exchange rate variance and the cost of intervention itself. It is expected that controlling exchange rate to move within optimal band will help support the objective of stabilizing the international trade sector and the whole economy as well.

In this paper, we say that the exchange rate intervention is beneficial whenever it can further help stabilize national output and price. On the contrary, the exchange rate intervention is regarded as a harmful operation if it further destabilizes (i.e., increasing the variance of) national output and price.

Mundaca and Øksendal (1998) employed a combined stochastic control and impulse control model to find optimal exchange rate intervention under target zone setting. They used two types of monetary policy tools, interest rate and foreign exchange market operations (buying and selling of foreign currency against national currency), to intervene in foreign exchange market in order to minimize total cost, which comprises the running cost (i.e., volatility of exchange rate) and the intervention cost. The result is that the system cannot be solved to get analytical solution. In addition, the numerical solution was not provided by the authors. On the other hand, Cadenillas and Zapatero (1999) used impulse control method to find optimal exchange rate intervention under target zone system. Buying and selling reserve was used as a tool of intervention. They were able to obtain numerical solution for optimal double band of exchange rate and the size of intervention was also derived.

Cadenillas and Zapatero (2000) extended their model by using two intervention tools, interest rate and direct buying/selling of foreign currencies. They employed both classical and impulse stochastic control methods to solve for optimal intervention under target zone system of exchange rate. They provided numerical solution to the problem, but yet again, the analytical solution was unsolvable.

Moreno (2007) extended the base-line model to allow for the volatility of the underlying process to be stochastic but mean-reverting. Numerical solutions were derived for many applications in economics and finance.

It is interesting to note that the existing literature analyzes the problem under partial equilibrium framework. Hence, the effects of exchange rate intervention on other sectors of the economy have not been taken into account.

In this paper, we will analyze the problem of exchange rate intervention under general equilibrium framework. Under this setting, the exchange rate intervention will have spillover effects, both benefit and cost, to the whole economy. In this connection, we introduce the "Degree of Openness" (using q as a symbol) to play the role as a linkage between the international trade sector and domestic sectors and hence the spillover effects of exchange rate intervention on domestic sectors of the economy can be analyzed.

3. The Degree of Openness

The degree of openness (q) is measured by the ratio of the value of international trade (export plus import) to the gross domestic product (GDP) or national income. That is,

$$\textit{Degree of Openness } (q) \quad = \quad (\textit{Export} + \textit{Import})/\textit{GDP}$$

We postulate that, with high degree of openness, the exchange rate intervention will be beneficial and hence more intervention is preferred. This is due to the fact that, with high degree of openness, the international trade transactions constitute large portion of GDP or national income. Therefore, stabilizing international trade sector via exchange rate intervention will also help stabilizing the whole economy as well.

On the contrary, with low degree of openness, the exchange rate intervention is costly or harmful and hence less intervention is preferred. This is due to the fact that the benefit from stabilizing international trade, which in this case constitutes only small portion of national income, may not be high enough to cover the cost arising from market distortion or loss of efficiency in domestic sectors due to exchange rate intervention itself.

We explore the degree of openness (using q as a symbol) of various countries and the associated exchange rate regime adopted for use by each country. By intuition, with high q (i.e., high proportion of international trade in the composition of gross domestic product (GDP)), we would expect that stabilizing international trade sector (through exchange rate intervention) will result in the stabilization of the whole economy as well. Therefore, with high q, we would conjecture that the central bank tends to intervene heavily in exchange rate, hoping that this will help stabilize the whole economy as well. On the contrary, with low q, it will not be beneficial to intervene in exchange rate because the benefit from intervention may not be high enough to cover the cost of intervention.

In our analysis here, the cost of intervention involves the distortion of domestic variables, such as money supply or interest rate, in such a way that they deviate from their optimal levels. These deviations will in turn causing the variations of national output and price.*

Below is the list of various countries, each is associated with the degree of openness (q) and the exchange rate system adopted for use in that country.

Table 1 Degree of Openness for Sample Countries and Exchange Rate System

Country	*Degree of*	Exchange Rate
UK	28	Free float
US	25	Free float
Japan	24	Free float
S. Korea	96	Free float
Brazil	20	Free float
Australia	39	Free float
Philippines	62	Free float
Singapore	380	Managed Float
Hong	380	Currency Board
Thailand	125	Managed Float
Malaysia	172	Managed Float
Indonesia	45	Managed Float
China	66	Managed Float

From the above information, we find that the countries with high value of q will tend to lean towards fixing or managed floating of exchange rate rather than free float. This may be due to the simple fact that, with high q, stabilizing international trade sector via exchange rate intervention will help stabilizing the whole economy (i.e., total GDP) as well. Examples of these countries are Singapore, Hong Kong, Thailand, and Malaysia.

For countries with low q (less open to trade), the international sector is small relative to domestic sector. Therefore, it can be deduced that, shifting domestic resources for exchange rate intervention may not be worthwhile since the cost of domestic resource distortion may outweigh the benefit derived from intervention in international sector. Hence, the exchange rate intervention will be harmful or not beneficial when q is small.

As can be seen from the table, most countries with low *q* (i.e., less open to trade) such as US, UK, Japan, Brazil, use a free float system of exchange rate. This might be due to the reason that, with low *q*, the intervention in exchange rate will destabilize the whole economy (i.e., causing domestic sectors to swing out of equilibrium as a result of disturbing domestic resource for use in exchange rate intervention).

4. Exchange Rate Intervention Model

In this section, the base-line model is introduced. This model is based on Cadenillas and Zapatero (1999) where they employed impulse control method to analyze the optimal intervention of exchange rate by the central bank. The set-up of the model is illustrated below. It will be used as a stepping stone for an extension in Section 5 when the managed floating regime is incorporated and the degree of openness is introduced.

Exchange Rate Process

Let (Ω, F, P) be a probability space equipped with a filtration $\{F_t\}$ representing a flow of information over time, with F_t being the information available at time t.

Without intervention from the central bank, the exchange rate process X_t is assumed to follow a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t , \tag{1}$$

or equivalently,

$$X_t = x + \int_0^t \mu X_s ds + \int_0^t \sigma X_s dw_s \tag{2}$$

Where

X_t is a stochastic process, $X > 0$, defined on (Ω, F, P) and adapted to $\{F_t\}$, with continuous sample path. It is measured in units of local currency per one unit of foreign currency.

x is the value of X at time $t = 0$

□ is the drift term

σ is the diffusion or volatility of X_t

W_s is the Weiner process

However, when the exchange rate is intervened by the central bank, the exchange rate process will evolve according to

$$X_t = x + \int_0^t \mu X_s ds + \int_0^t \sigma X_s dw_s + \sum_{i=1}^{\infty} I_{\{\tau_i < t\}} \xi_i \qquad (3)$$

Where the last term of (4.3) represents the sequence of intervention

T_i is the time at which the ith- intevention has taken place

ξ_i is the size of the intervention (measured in term of ΔX).

I is the indicator variable. I = 1 when the central bank intervenes. It is zero otherwise.

The Impulse Control

The impulse control is a pair

$$U = (T, \xi) = (T_0, T_1, T_2, \ldots, T_n, \ldots; \xi_0, \xi_1, \xi_2, \ldots, \xi_n, \ldots) \qquad (4)$$

Where

$T_0 < T_1 < T_2 < \ldots < T_n$ is a sequence of the intervention (stopping time).

ξ_i is a sequence of random intervention size (or simply called, the impulse). $\xi_i : \Omega \rightarrow R$ is F- measurable. The central bank intervenes on the strong side of exchange rate by adding the quantity $\xi > 0$ (measured in term of ΔX) to exchange rate process in order to weaken the local currency. On the other hand, on the weak side of exchange rate, the central bank intervenes by adding the quantity $\xi < 0$ to exchange rate process in order to strengthen the local currency.

In this baseline model, the purpose of the intervention by the central bank is to keep the exchange rate to fluctuate within the desirable band around the pre-determined target (which is set exogenously by the central bank) in order to minimize the variance of exchange rate, given the cost of intervention.

The Cost Functional

Analytically, the central bank will select the control pair (T, ξ), the timing and the size of intervention respectively, in order to minimize total cost functional *J*. The total cost comprises the running cost and the intervention cost. The running cost is the cost arising from the fluctuation or the variance of exchange rate process. The intervention cost is given exogenously.

While the running cost is decreasing with the intensity of interventions, the intervention cost will be increasing. Therefore, under certain conditions to be discussed later, the two costs combined will form the total cost function that is convex and hence has a minimum (given that it is smooth and continuous).

$$J(X,T,\xi) := E\left[\int_0^\infty e^{-rt} f(X_t)dt + \sum_{n=1}^\infty e^{-r\tau_n} g(\xi_n) I_{\{\tau_n<\infty\}}\right] \tag{5}$$

Where *E* is the expectation operator based on information available at time 0.

r is the discount factor. The first term in the objective function is the running cost (i.e., the cost of exchange rate uncertainty arising from its fluctuations). It can take the form:

$$f(x) = (x-\theta)^2 \tag{6}$$

Where θ is the exchange rate target determined exogenously by the central bank.

The second term in the cost functional represents the intervention cost. It can be broken down into fixed cost and proportional cost. Furthermore, the intervention is divided into intervention on the weak side (ξ < 0) and intervention on the strong side (ξ > 0) of the exchange rate process.

$$g(\xi) = \begin{cases} C + c\xi & \text{if } \xi > 0 \text{ (strong-side intervention)} \\ 0 & \text{if } \xi = 0 \text{ (no intervention)} \\ D + d\xi & \text{if } \xi < 0 \text{ (weak-side intervention)} \end{cases} \tag{7}$$

C, D, c, and d is constants, each greater than zero. ξ is measured in term of ΔX.

<u>The Value Function:</u>

According to Cadenillas and Zapatero (1999), we denote *V* as the value function:

$$V(x) := \inf_{A(x)} \{J^v(x;T,\xi);(T,\xi) \in \mathrm{A}(x)\} \tag{8}$$

Where,

A(x)	is the set of admissible control
$U = (T,\xi)$	is admissible control, $v \in A(x)$
J^v	is the cost function associated with each admissible control

The *minimum cost operator* or *intervention operator* is defined by:

$$MV(x) := \inf\{V(x+\eta) + g(\eta) : \eta \in R, x+\eta \in (0,\infty)\} \tag{9}$$

The *MV(x)* represents the total value after each intervention, comprising the value of the process after intervention plus the intervention cost g(.). Note that it is optimal to intervene whenever

$$MV(X) = V(X) \tag{10}$$

That is, the central bank will intervene when *X* hits either side of the optimal exchange rate band. Therefore, our optimization problem boils down to finding the optimal exchange rate band with the optimal points at which the exchange rate process will restart after each intervention. Let $\phi(x)$ be the test value function which is the candidate solution for optimality condition to be explained below.

The *infinitesimal operator* is defined as

$$\Im\phi(x) := \frac{1}{2}\sigma^2 x^2 \frac{d^2\phi(x)}{dx^2} + \mu x \frac{d\phi(x)}{dx} - r\phi(x) \tag{11}$$

Equation (11) represents the value function when it is optimal not to intervene (i.e., to leave the process X to move independently within the band). Then we will have in a non-intervention (or continuation) region

$$\Im\phi(x) + f(x) = 0 \qquad \text{for } a < x < b \tag{12}$$

Where $f(x) = (x-\theta)^2$. Equation (12) is called a *Feynman- Kac equation*.

The Quasi Variational Inequalities (QVI)

The QVI is the optimality condition for the impulse control problem. It can be shown that the problem of choosing the best strategy (*T*, ξ) in order to minimize cost functional is equivalent to solving the problem of QVI.

We say that a function ***V***: *(0, ∞)* ⟶ *(0, ∞)* satisfies the QVI if for every x $\in$ (0, ∞),

$$\Im v(x) + f(x) \geq 0 \tag{13}$$

$$v(x) \leq Mv(x) \tag{14}$$

$$[v(x) - Mv(x)][\Im v(x) + f(x)] = 0 \tag{15}$$

The solution ***V*** of the QVI separates the interval (*0, ∞)* into two disjoint regions:

Continuation Region (CR) where it is optimal not to intervene

$$CR = \{x \in (0,\infty) : v(x) < Mv(x) \text{ and } \Im v(x) + f(x) = 0,$$

and, *Intervention Region* (IR) where it is optimal to intervene

$$IR = \{x \in (0,\infty) : v(x) = Mv(x) \text{ and } \Im v(x) + f(x) > 0\}$$

Therefore, the solution to the QVI will be in the form of optimal band with optimal restarting point(s) for the process X to restart after each intervention.

The Solution to QVI

We conjecture that the solution *(T*, ξ*)* as characterized by the band with parameters *a* and *b* as the boundaries and α and β as restarting points after each intervention, can be solved from the six equations below. The six unknowns to be solved for are *A, B, a, b, α, and β.* (Cadenillas and Zapatero, 1999).

$$h(a) = h(\alpha) + C + c(\alpha - a) \tag{16}$$

$$h(b) = h(\beta) + D + d(b - \beta) \tag{17}$$

$$h'(a) = -c \tag{18}$$

$$h'(b) = d \tag{19}$$

$$h'(\alpha) = -c \tag{20}$$

$$h'(\beta) = d \tag{21}$$

Where

$$h(x) = Ax^{\gamma_1} + Bx^{\gamma_2} + \left(\frac{1}{-\sigma^2 - 2\mu + r}\right)x^2 - \left(\frac{2\theta}{r - \mu}\right)x + \frac{\theta^2}{r} \tag{22}$$

which is a solution of the Feynman- Kac equation in (22). We then have

$$h'(x) = A\gamma_1 x^{\gamma_1 - 1} + B\gamma_2 x^{\gamma_2 - 1} + \left(\frac{2}{-\sigma^2 - 2\mu + r}\right)x - \left(\frac{2\theta}{r - \mu}\right) \tag{23}$$

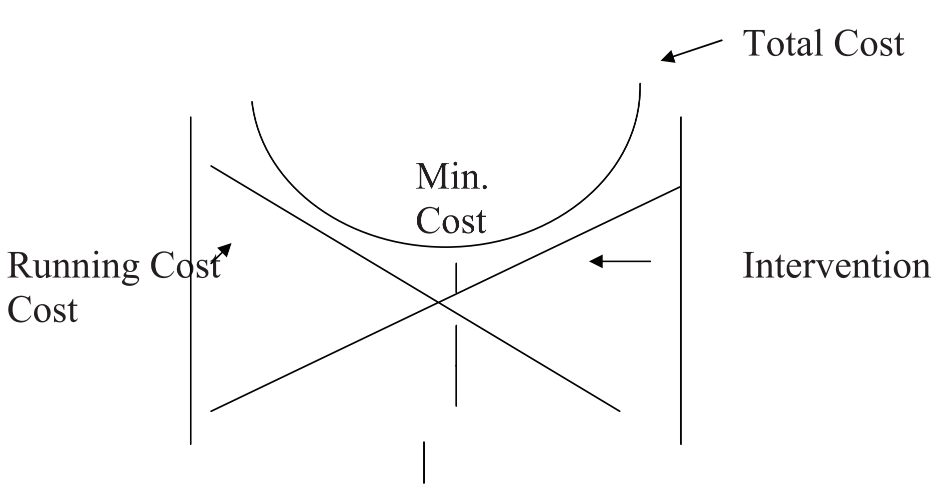
Verification of the solution

Generally, the impulse control problem cannot be solved to get analytical solution. Therefore, we have to solve for numerical solution instead. In this regard, we need to verify that the numerical solution so obtained is indeed optimal. That is, suppose that $\phi(x)$ is the solution of QVI, where $\phi(x)$ can be regarded as a test function. Then we need to prove that $\phi(x) = J(x) = V(x)$. The proof of the Verification Theorem is provided by Cadenillas and Zapatero (1999).

Graphical Illustration

The simple graph below explains the optimal exchange rate intervention. For intuitive exposition purpose, we assume that the total cost function is continuous and convex so that that the minimum can be obtained. The central bank chooses the size of the band, representing the level or intensity of intervention in order to minimize total cost.

Figure 1: Optimal Exchange Rate Intervention.



Numerical Example

Let μ = 0.1*, σ = 0.3*, *r* = 0.06, θ = 30, C = 1, D = 1,

Notice that μ = 0.1 means there is a depreciation pressure on Thai baht. For ease of computation, we assume there are only fixed costs C and D.

Using command “FindRoot” of software program Mathematica, the solutions are obtained as follow:

$$a = 25.40,\ \alpha = \beta = 29.79,\ b = 34.78$$

$$A = -453{,}356,\ B = -5{,}196$$

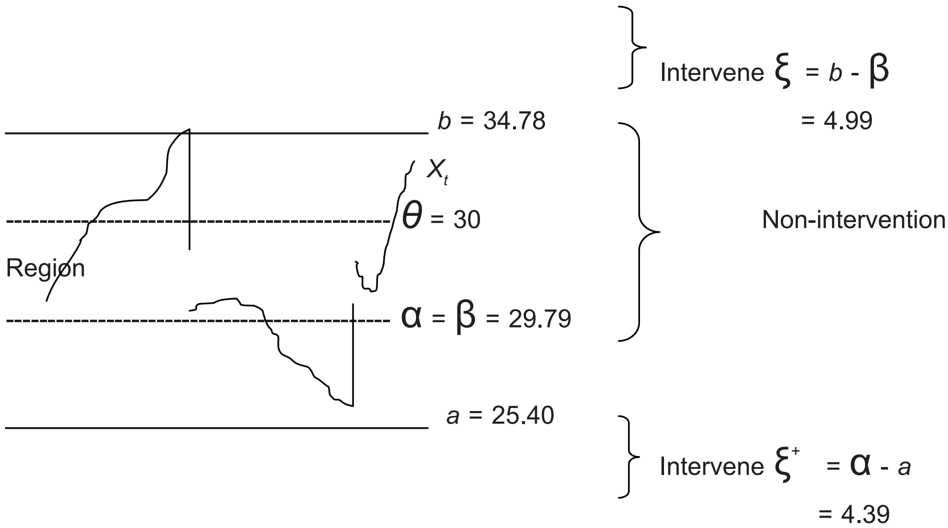
Note that the optimal values of the starting points α and β are equal at 29.79 baht/US dollar in our case. However, it is still left open that in some conditions the values of α and β may be different from each other.

The estimates of parameter values $\hat{\mu} = 0.1$ and $\hat{\sigma} = 0.3$ for Baht/USD are similar to what they use in modeling GBM for exchange rate in current theoretical papers. The details of the estimation can be obtained from the author upon request.

From the solutions, we will have the optimal rule as follow:

- When *x* hits the lower boundary at *a* (= 25.40), intervene with the size α - *a* (= 29.79 – 25.40 = 4.39) to bring *x* back to restart at point α.
- When x hits the upper boundary at *b* (= 34.78), intervene with the size *b* - β (= 34.78 – 29.79 = 4.99) to bring *x* back to restart at point β.
- No intervention when *x* moves within the boundaries (*a, b*).

Figure 2: **Baseline Model- Optimal Exchange Rate Band**



Note that the optimal band in the baseline case, where there is no monetary policy involved, is very wide that it may look unreasonable in practice. Notice that the restarting point $\alpha = \beta = 29.79$ lies below the target exchange rate. This is to compensate for the depreciation pressure as indicated by a positive drift $\mu = 0.1$.

1. **Extended Model: Incorporating the Degree of openness**

In this section, we incorporate the degree of openness (*q)* that represents the spillover effect of exchange rate intervention itself on domestic economy. It is expected that with high *q*, the intervention to reduce exchange rate variance will have spillover effect in reducing the variance of national output and price as well. This is due to the fact that, with high *q*, large portion of national output (i.e., GDP) is accounted for by international trade sector activities. Hence, stabilizing international trade sector via exchange rate intervention will also help stabilizing national output and price as well.

Vice versa applies for the case of low degree of openness where the exchange rate intervention will have a negative spill over in that it will ultimately result in disturbing domestic economy rather than stabilizing it.

The Cost Functional

The weight *1/q* is inserted to the cost function to reflect the influence of the degree of openness on the cost of intervention. In particular, when the degree of openness (*q*) is high, the intervention cost (*1/q*) will be low, and hence it will be optimal to intervene more. The opposite holds for the low degree of openness.

Formally, the central bank chooses the control pair (*T,* ξ) to minimize the total cost

$$J(X,T,\xi) := E[\int_0^\infty e^{-rt} f(X_t)dt + \sum_{n=1}^\infty e^{-r\tau_n} 1/q.g(\xi_n) I_{\{\tau_n<\infty\}}], \tag{24}$$

where

q is the degree of openness, where $q \in (0,\infty)$. It can be thought of as a weight of spillover effect (benefit or cost) of exchange rate intervention. The higher the value of *q*, the higher the benefit to be obtained from the intervention and hence the cheaper will be the cost of intervention relative to the running cost. Consequently, the central bank will tend to move toward more intervention or more fixing of X. On the other hand, the lower the value of the *q*, the more expensive will be the cost of intervention relative to running cost and the central bank will tend to move toward more floating of X. We will normalize the weight of the running cost to equal to one.

Therefore, *1/q* can be thought of as an index of spillover effect (benefit or cost) of exchange rate intervention. Note that, *q* may be entered into the cost function in a more sophisticated manner. But that will be relegated to future attempt.

Finding Numerical Solution

As before, we conjecture that there exists an optimal solution (T*, ξ*) characterized by four parameters *a*, α, β, *b* with $0 < a < \alpha \leq \beta < b < \infty$. There are six unknowns, A, B, *a, b*, α, and β, to be solved from six equations below. The solutions *a**, *b**, α*, and β* will characterize the optimal intervention in terms of optimal exchange rate band (a^*, α^*, β^*, b^*) and the optimal size (ξ) of intervention equals (α^* - a^*) and (b^* - β^*).

$$h(a) = h(\alpha) + (1/q)\{C + c(\alpha - a)\} \tag{25}$$

$$h(b) = h(\beta) + (1/q)\{D + d(b - \beta)\} \tag{26}$$

$$h'(a) = -(1/q)c \tag{27}$$

$$h'(b) = (1/q)d \tag{28}$$

$$h'(\alpha) = -(1/q)c \tag{29}$$

$$h'(\beta) = (1/q)d\ , \tag{30}$$

Numerical Example

Let θ = 30, σ = 0.3, r = 0.06, μ = 0.1,
C = 1, c = 0, D = 1, d = 0, q = 125.

Here, we suppress fixed cost $C = D = 1$, and proportional cost $c = d = 0$ in order that we can focus on the impact of the degree of openness (q) where q = 125 is the value of the degree of openness for Thailand. We postulate that, with high q, stabilizing exchange rate (by reducing its variance) will further help stabilize domestic real output and inflation as well (positive spillover effect of the intervention). Consequently, we would expect the exchange rate band to be narrow under high q.

Using FindRoot command of Mathematica software program, we obtain the solution for the above 6 non-linear equations as follow:

a = 28.61, $\alpha = \beta$ = 29.98, b = 31.41
A = - 473,161, B = - 5,204

Comparing results:

Parameter	*Baseline model*	*With managed floating*
a	25.4	28.61
α=β	29.79	29.98
b	34.78	31.41
(b-a)	9.38	2.8
(b-θ)	4.78	1.41
(θ-a)	4.6	1.39

As expected, under managed floating regime, coupled with the high value of the degree of openness (*q*), the size of the band (*b* - *a*) gets smaller as compared to that of the baseline model, signifying more intervention by the central bank in order to reduce the variance of exchange rate.

Model Simulations

Varying the *q* (the weight representing the degree of openness), other parameters held constant: (μ = 0.1, σ = 0.3, r = 0.06, θ = 30, C = 1, D = 1)

q	*(b – a)*	*(b – θ)*	*(θ – a)*	*(α = β)*
1	34.78 – 25.40 = 9.38	4.78	4.6	29.79
2	34.01 – 26.12 = 7.89	4.01	3.88	29.85
4	33.37 – 26.73 = 6.64	3.37	3.27	29.9
8	32.82 – 27.24 = 5.58	2.82	2.76	29.93
16	32.37 – 27.67 = 4.70	2.37	2.33	29.95

32	31.97 – 28.06 = 3.91	1.97	1.94	29.97

As expected, the larger the value of q (the degree of openness used as a proxy of the spillover effect of the intervention), the smaller will be the size of the band ($b - a$), signifying more intervention by the central bank. This means that, for the case of high q, exchange rate intervention is beneficial and the central bank will tend to intervene more in foreign exchange market.

6. Summary

In this paper, we attempt to incorporate the managed floating policy into the standard exchange rate intervention model. The degree of openness is introduced as a proxy of the spillover effect of the intervention itself. The impulse control method based on Cadenillas and Zapatero (1999). The model is applied to the case of Thai Baht/US dollar exchange rate.

The analysis is carried out under general equilibrium framework so that the spillover effects from exchange rate intervention on other sectors of the economy can be taken into account. Therefore, under this setting, the exchange rate intervention can be either beneficial or harmful, depending on the degree of openness of the country under consideration. With high degree of openness for the case of Thailand ($q = 125$), the optimal exchange rate band gets smaller as compared with that of the baseline case, signifying more exchange rate intervention by the central bank.

The result can be explained intuitively as follow. With high q (the degree of openness) as for the case of Thailand, significant portion of total national output and price is accounted for by international trade transactions. Therefore, the exchange rate intervention that is aimed towards stabilizing international trade sector will further help stabilizing the whole economy as well. Therefore, exchange rate intervention is beneficial for the case of high q. It can be further deduced that, for the case of low q, the international trade transactions constitute insignificant portion of national output. Consequently, less intervention is preferred due to the fact that the benefit from intervention is too small that it may be outweighed by intervention cost. The finding will be useful for the central bank to design the exchange rate intervention program that will result in an attainment of the second-best efficiency.

Finally, the future direction of the research in this area is proposed as follow. The model used in this paper can be extended to include inflation targeting monetary policy. Under this setting, we can analyze the interactions between the managed floating regime and inflation targeting monetary policy which will lead to the optimal exchange rate band under various scenarios of interest. Another area of research may involve the estimation of the effect of exchange rate intervention (spillover effect) on the variances of national output and price. The estimate so obtained can then be used as an evidence-based parameter to replace the degree of openness which is used in this paper. Certainly, such estimate contains empirical information and should give us more confidence than using the degree of openness.

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