The Optimal Parental Time for Thai Economy

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Abstract

Since parents invest both their material resources and time into raising their children, this paper constructs a five period Heterogeneous Overlapping Generation model to investigate the optimal parental time allocation for their children. We use the calibrated parameters to find the optimal parental time for Thai economy by differing values of parental time in the model to generate the results for three different scenarios. To minimize wealth inequality the parental time should be set at 12-15 percent. To maximize the total output, the average parental time should be set at 12-21% of available time. While increasing the parental time will improve output and wealth equality,

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บทความที่เหมาะสมในการเลี้ยงดูบุตรสำหรับเศรษฐกิจไทย

บทคัดย่อ

พ่อแม่ทุ่มเททรัพยากรและเวลาเพื่อเลี้ยงดูบุตรย่อมส่งผลต่อพัฒนาการของลูกทางศึกษาที่มีสำหรับวิธีการที่เหมาะสมและเหมาะสมทางเศรษฐกิจไทย ผลการศึกษาพบว่า ถ้าลูกมีความใช้เวลาในการเรียนรู้แล้วจ้างพ่อแม่ควรจะมีการจัดสรรเวลาในการเลี้ยงดูบุตรประมาณร้อยละ 12-15 ของเวลาที่มีอยู่ทั้งหมด ในขณะที่ถ้าต้องการให้ระดับของผลผลิตประชาชาติสูงสุดพ่อแม่ควรจะมีการจัดสรรเวลาในการเลี้ยงดูบุตรประมาณร้อยละ 12-21 ของเวลาที่มีอยู่ ผลการศึกษาสรุปว่า การสละเวลาในการเลี้ยงดูบุตรของพ่อแม่จะมีผลต่อการเจริญเติบโตทางเศรษฐกิจและการลดความเหลื่อมล้ำของประเทศได้ด้วยกัน

ค่าสำคัญ: เวลาเลี้ยงดูบุตร, เศรษฐกิจไทย, ความเหลื่อมล้ำ

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1. Introduction

In altruism hypothesis, parents care about the well-being of their children. They invest both their material resources and time into raising their children because they gain utility from such behaviors. Many economic literatures have focused on the material part of investment in children, such as education expenditure and bequest. However, time investment is thought to be equally important to the development of children’s skills and abilities. Casarico and Sommacal (2012) point out that time spent with children plays a large role in determining human capital achievements. In their concept, the intergenerational time transfer by a parent can affect basic cognitive and behavioral skills of a child, which in turn will have an impact, together with formal schooling, on his human capital and on his productivity. If parental time investment on children is one of key factors in enhancing children’s human capital later in life and if, as it is recognized in the growth literature, human capital is a key engine for growth, the parental time allocation between paid work, leisure and childcare might have crucial implications for economic growth and development.

Similar to other resources, the amount of time invested in children is determined by parents’ budget constraint and their preferences. The parent’s time devoted to childcare is taken away from leisure and/or paid employment, presenting an opportunity cost. Parents consider the benefits and costs of time spent on activities and make decisions accordingly in such a way that maximizes utility. However, like other resources, their decisions on time allocation are based on private returns and costs. From the macroeconomic perspective, these parental time allocation decisions in aggregate affect an economy directly through labor supply. Spending more time with children may enhance children’s human capital, hence increases their productivity and efficient wage in the future. However, it comes with a cost, for working parents must trade their working time with the time spent on children. A social planner would choose an amount of parental time for childcare such that reaches a social objective. This social parental time may not coincide with the parents’ decisions in competitive economy. This draws our motivation to find out what the social parental time would be in order to achieve social objectives, in particular, to maximize the total output and to minimize the inequality.

To the best knowledge, there are no papers exploring intergenerational transfer of time in the heterogeneity model. Therefore, the first part of this study introduces an altruistic parental time economic model that parents can allocate their time over leisure, working and childcare, expecting that giving more time on childcare will result in their children higher efficient wages in the future. The same as giving proper education, this is a human capital development extension. The proposed model is calibrated to Thai economy. Then the results of the model will be compared to the features of the Thai economy from a previous study. Having already established
the parental time channel in the model, the second part of the study deals with the parental time as an exogenous variable in the model. The model has been tested through different values of parental time to find solutions for the Thai economy regarding different social goals such as minimizing earning and wealth inequality and maximizing total output.

2. Literature Review

Most theoretical studies of inequality and other macroeconomic variables are based on general equilibrium quantitative models with heterogeneous agents. The literature review employs general equilibrium quantitative models to mimic an economy and to explain inequality. The models are categorized as life-cycle and intergenerational transfer (involuntary transfer or accidental bequest and altruism) by motives of saving and transfers.

Life cycle model

In a life-cycle related model, it is commonly assumed that an individual cares about himself. He only makes a saving to smooth his consumption over the life time. In this motive there is no altruistic monetary transfer linkage between individuals from different generations. Among academics studying inequality with a life-cycle hypothesis, Aiyagari (1994) modified a life-cycle model to involve uninsured idiosyncratic shocks. While the author deviates from a standard representative agent model by including a large number of heterogeneous agents, the aggregate variables are unchanged. With a combination of incomplete market and borrowing constraint, agents save in order to smooth their consumption in the face of uncertain individual labor incomes. The author compares income and wealth distribution generated from his model with those of the U.S. data. The model cannot generate the observed degrees of inequality. Keeping the same features of heterogeneity, idiosyncratic shocks of income, incomplete market and borrowing constraints in the model, Huggett (1996) adds new features such as longevity uncertainty, income taxation and social security to improve the match of the U.S. wealth distribution. His model economies with these features are able to replicate measure of both the aggregate wealth and the transfer wealth in the U.S. economy. However, the model economies examined do not generate all of the concentration of wealth in the upper tail of the distribution. In particular, the model economies generate only about half of the wealth held by the top 1 percent in the U.S. In the same line of studies, Hendricks (2004) shows that life-cycle models have difficulties accounting for empirical relationship between lifetime earnings and household wealth. Quantitative life-cycle implies a far tighter relationship between earnings and wealth than is observed in U.S. data. As a result, life-cycle models understate wealth inequality among households with similar lifetime earnings. These findings suggest that a standard life-cycle theory fails to account for an important source of wealth inequality. This may be because households have neither the incentives nor the time to accumulate sufficiently large amounts of wealth. They neglected
parent-child link which may be a main incentive of wealth accumulation that accounted for the wealth inequality of the real data.

**Involuntary transfers**
The involuntary transfer hypothesis has been proposed as one of the causes of intergenerational transfers when a slow decumulation in retirement age which does not accord with life-cycle hypothesis was noticed in the U.S. Davies (1981) finds evidence that the slow dissave is not caused by intentional bequest motive but to a large extent by lifetime uncertainty. With imperfect private insurance markets, retired parents who have to cope with lifetime uncertainty may leave considerable accidental or precautionary bequests. Presumably less well-off individuals are more likely to save more during retired age and do not either want to trade with their children or make inter vivos transfers. Among the studies on involuntary bequest’s effects on the U.S. wealth inequality, Gokhale et al. (2000) constructs an overlapping generation model with uncertain lifespan. Agents have life-cycle preferences with no intentional bequest motive. They leave bequests only because their resources are not fully annuitized. In their model, inequality in inheritances can be influenced by many factors such as earnings inequality, transmission of earnings inequality across generations, number and spacing of children, heterogeneous rates of return, time preference, annuitization of retirement savings through social security, and progressivity of income tax system. The model generates a distribution of wealth that closely approximates the degrees of inequality and skewness in the actual U.S. data.

**Altruism**
The altruism hypothesis emerged as a clear cut alternative to self-interest, normally assumed to prevail on the market. A large number of literatures about altruism have been influenced by the work of Becker (1974) and Becker(1991). In Becker’s model of altruism, parents care about the well-being of their children, using bequests and other gifts to obtain the desired distribution of resources within the family. When the pure altruism is operative the current generations are connected to future generations by a chain of intergenerational transfers. The model leads to Ricardian equivalence (Barro, 1974). Many subsequent studies incorporate altruistic behavior in the models. For example, Kotlikoff and Summers (1981) use historical U.S. data to directly estimate the contribution of intergenerational transfers to aggregate capital accumulation, and Becker and Tomes (1979) study intergenerational mobility, human capital investment and inequality. Among the earlier partial equilibrium, quantitative studies, Davies (1982) analyses the effects of various factors, including bequests, on economic inequality in a one-period model without uncertainty.
Subsequent studies on income and wealth inequality include altruistic motive. For example, Laitner (2001) mixes life-cycle and dynastic behavior. In his model, all agents save for life-cycle purposes, but only some of them (a fraction $\lambda$ of population) care about their own descendants. Such agents may choose to accumulate estates for bequests. Non altruistic families care solely about their own lives. In the model, there are no precautionary savings. The author shows that the concentration in the upper tail of the wealth distribution generated from the model can match the real data by choosing the fraction of households that behaves as a dynasty.

Similar to Laitner (2001), Castaneda et al (2003) also mix the main features of the life-cycle and of the dynastic hypothesis. However, without introducing a fraction of each type in an economy, they assume that all households in the model economies are altruistic and that they go through the life-cycle stages of working age and retirement. These households, with identical and standard preferences, receive an idiosyncratic random endowment of efficiency labor units. They do not have access to insurance markets and save in part to smooth their consumption. The households also save to supplement their retirement pensions and to endow their estates. The authors find that their model economy does a very good job of accounting for the U.S. distribution of earnings and wealth. In particular, they report the Gini index for their distribution of earning and wealth as 0.63 and 0.79 respectively, compared to those of 0.63 and 0.78 in the U.S. data. The share of earnings and wealth of the top 1 percent of households are 14.93 and 29.85 percent respectively in the model which almost exactly match the 14.76 and 29.55 percent in the U.S. data.

Nishiyama (2002) extends a standard heterogeneous agent overlapping generation model by adding two-way intergenerational altruism, lifetime uncertainty, a fertility shock, and borrowing constraints. In the model, households in the same dynasty play a Nash game in each period to determine their optimal consumption, working hours, inter vivos transfers, and savings. The model suggests that when deciding the level of bequests, a parent household considers the future utility of its child households, on average, about 20 percent less than the amount it considers its own future utility. But, the parent household’s motive for inter vivos transfers is much weaker than its motive for altruistic bequests. Although the model replicates the wealth distribution of the United States fairly well in term of the Gini coefficient of wealth distribution, the share of wealth of the top 1 percent of households is 14.6 percent in the model which is lower than the 29.6 percent in the data. He concludes that the effects of bequests and inter vivos transfers on wealth distribution in the model are not very large.
Mixtures of involuntary transfer and altruism

There are a large number of literatures that mix some features of involuntary transfer and altruistic transfer in a model. Among them, Heer (1999) develops an overlapping generation model in which heterogeneous agents face uncertain lifetime and leave both accidental and voluntary bequests to their children. Furthermore, agents face stochastic employment opportunities. The results indicate that bequests only account for a small proportion of observed wealth heterogeneity. The author suggests that neglecting productivity heterogeneity within generations in the model may be one of the reasons why the endogenous wealth heterogeneity of his model is smaller than the one observed empirically. He considers that adding a channel for transfer human wealth may also improve the result.

Ocampo and Yuki (2006) investigate the quantitative importance of different savings motives on the distributions of wealth and consumption and aggregate capital accumulation. In their heterogeneous overlapping generation model, agents differ in age, ability, earnings shocks, and inherited bequests. They also assume that there are uninsurable idiosyncratic risks associated with uncertain lifetime and the earnings shocks. The authors find that, in the baseline model, the top 1 percent and 5 percent hold wealth 24.17 percent and 49.78 percent respectively, compared with 29.55 percent and 53.5 percent in the U.S. data.

As suggested by Heer (1999) that transferring human capital may be as important as transferring the physical capital in explaining inequality, De Nardi (2004) focuses on the transmission of both physical and human capital from parents to children. The author adopts a computable, general equilibrium, incomplete-markets, life-cycle model in which parents and their children are linked by bequests, both voluntary and accidental, and by the transmission of earnings ability. The author finds that voluntary bequests can explain the emergence of large estates, and characterize the upper tail of the wealth distribution in the data. Accidental bequests alone, even if unequally distributed, do not generate more wealth concentration. The presence of a bequest motive also makes lifetime saving profiles more consistent with the data. A human capital link, through which children partially inherit the productivity of their parents, generates an even more concentrated wealth distribution.

With the similar assumptions made on the model in De Nardi (2004), Yang (2005) adds a borrowing constraint into his model. The households save to self-insure against labor earning shocks and life-span risk, for retirement, and possibly to leave bequests to their children. The members of successive generations are linked by bequests and by the children’s inheritance of part of their parent’s productivity. The households do not know the exact time and amount
of inheritance, and they are not allowed to borrow against its future neither. The existence of a borrowing constraint prevents households from smoothing consumption inter-temporally. The result confirms findings in De Nardi (2004), that a model without intergenerational links cannot generate a skewed wealth distribution comparable with the data. The Gini coefficient of wealth is only 0.64, compared with 0.72 in the benchmark economy and 0.78 in the data.

**Educational bequest**

Early studies on income and wealth inequality seem to agree that intergeneration link, both physical wealth and human wealth, contributes to distribution and concentration in the upper tail of wealth distribution in the U.S. These works take earnings or wages as an exogenous random process and then proceeds to characterize the distributional implications of optimal consumption-savings and labor-leisure behavior. Huggett (2006) argues that this research agenda should integrate deeper foundations for the determinants of earnings and wages into these models by allowing earnings to be endogenous. In his model, each agent is endowed with some immutable learning ability and some initial human capital. Each period, an agent divides available time between market work and human capital production. Human capital production is increasing in learning ability, current human capital, and time allocated to human capital production. An agent maximizes the present value of earnings, where earnings in any period are the product of a rental rate, human capital, and time allocated to market work. The author establishes that the earnings distribution dynamics documented from the U.S. data can be replicated quite well by the model from the right initial distribution.

Chaisrisawatsuk (2004) considers an alternative hypothesis that may account for inequality in income and wealth distribution via another persistence channel. The author argues that parents’ concern for their children may be revealed during the parents’ lifetime in the form of paying for a given education level, rather than after death as occurs in planned bequests. By paying for a child’s education, parents can influence the likely level of earnings of a child. The author allows households to be subject to idiosyncratic risk. The wage income depends on an individual’s age, education and idiosyncratic shock. The level of education has impact on both the level of earning and the probability of the level of earning. The author compares the results generated by the model to the 2000 Household Socio-Economic Survey (HSES) produced by the National Statistical Office of Thailand. He finds that the model generates an education distribution closely similar to what is observed in the data. Furthermore, the earning and income distribution of the model matches well with the data. He concludes that intentional bequest, education as planned bequest, and the differences in idiosyncratic earning shocks are the important factors in explaining earning and income inequality in Thailand.
Most literature agrees that the inclusion of altruistic human capital development in an economic model can improve the results in explaining inequality. Recent works focus on the models with education as a mean to develop human capital when attempting to explain inequality. This research extends from the existing economic models by adding parental time as another intergenerational link to capture the parents’ behaviors and impact of parental time investment on childcare.

3. Environment of the Model

Households

Individuals in this economy live for five periods with certainty. In the first period, individuals known as children live with their parents. They do not make any economic decisions. Parents are to decide how much their children can consume, how much time they spend rearing their children and what level of education they should have, through altruistic motive. In the second period, individual known as young adults leave their parents and live on their own. They receive bequests left for them altruistically by their parents at the beginning of the period. They are endowed with a unit of time in which they decide how much time they spend on working and leisure. They also make decisions on their consumption and saving. In the third period, parents raise a number of children where the number is randomly drawn. Parents are endowed with a unit of time in which they decide how much time they spend on working, leisure and rearing children. They also make the decisions about their own consumption, saving, their children’s consumption, education level and bequests. In the fourth period, Old adults live without children since their children have left the family. Old adults still work and spend time not working with leisure. They make decisions about their own consumption and saving. The last period of life is the retirement period. Individual live without working, consume all what they have saved earlier and die at the end of the period.

There are two random events in the model that have significant effects on individuals. The first random event is an idiosyncratic shock which occurs at the beginning of each period. This shock differently affects individual’s earning depending on his own level of education and the amount of rearing time he gets from his parents in the first period. An agent with higher education will have a better chance of getting a better shock than the one with a lower education. The second random event is a number of children. At the beginning of the parents’ period, parents will be randomly drawn for a number of children which is from 1 to 5.

A part of saving the agents make is from life-cycle motive, to smooth their consumption over the life time. An assumption is that agents in this model are altruistic towards their children. The intergenerational transfer occurs only in the third period when the parent agents are altruistic
toward their children. Parents’ utility increase when their children utility and expected utility in the future periods increase. The children’s utility arises directly from the consumption and expected future incomes which in turn depend on education level, amount of time spent with parents and also the bequest given to the children.

In each period an agent has to face different economic environments which depend on the asset or wealth position, the education level, the time that his parents take care of him, the idiosyncratic earning shock, the number of children in the household. There are three levels of education, primary school, high school and university. There are 5 levels of parental time that parents can choose from to take care of their children. Given that parent are endowed with a unit of total time, fractions of time: 0.03, 0.06, 0.12, 0.18 and 0.24 respectively, spent with a child. The household utility function is of the constant relative risk aversion (CRRA) type (See Appendix A).

The household problem can be summarized in general form as follows: (See Appendix A for detail equations and solutions)

\[
v(j, a, dt, e, z, nc) = \max_{\{c,wt,a',c^k,dt,c,ecn\}} \left\{ \left[ \left( (1 - I_h)(1 - wt - I_{nc}n_cdt_c) \right) + I_{nc}b(n_c)n_cu(c^k) + I_{nc}b(n_c)n_cE[v^{\beta}(j,a,dt,c,e, z',n_c)|z] \right) + (1 - I_h)(1 - I_{En_c})E[v(j + 1, a', dt, e, z', n_c = 0)|z] \right) + I_{En_c}E[v(j + 1, a', dt, e, z', n_c)|z] \right\} \]

subject to

\[
c + I_{nc}n_c c^k + (1 - I_h)a' + I_{nc}n_c TC(e_c) + I_{nc}n_c beq \leq (1 - I_h)wn(j, dt, e, wt, z) + (1 + r)a \]

where the indicator variables are:

\[
I_h = \begin{cases} 
0, & \text{if } j \leq 5 \\
1, & \text{if } j > 5 
\end{cases} 
\]

\[
I_{nc} = \begin{cases} 
0, & \text{if } j \neq 3 \\
1, & \text{if } j = 3 
\end{cases} 
\]

\[
I_{En_c} = \begin{cases} 
0, & \text{if } j \neq 2 \\
1, & \text{if } j = 2 
\end{cases} 
\]
Where

- \( j \) is the period considered.
- \( a \) is an asset position an individual has at the beginning of the period.
- \( a' \) is an asset position an individual decides to have (saving) at the end of the period.
- \( dt \) is the parental time an individual receives when he is a child.
- \( e \) is an education level of an individual.
- \( z \) is an idiosyncratic earnings shock which individual receives at the beginning of the period.
- \( z' \) is an idiosyncratic earnings shock which individual receives at the beginning of the next period.
- \( n_c \) is a number of children.
- \( wt \) is working time
- \( c \) is a consumption level.
- \( \beta \) is the discount factor.
- \( E \) is the expectation operator.
- \( w \) is wage.
- \( beq \) is bequest.
- \( 1 + r \) is the interest rate.
- \( v \) is value function of an individual.
- \( v^c \) is value function of each child.
- \( c^k \) is the consumption level of a child.
- \( dt_c \) is the parental time level that a parent choose to rear a child.
- \( e_c \) is the education level of a child.
- \( TC(e_c) \) is the cost of education.
- \( b(n) \) is a discount factor with respect to the number of the children.

Firms

There are two goods in the model. In one market, a representative firm produces commodity goods while the other produces education. Both markets are assumed to be competitive. The production function of both types of firms is of the standard Cobb-Douglas production function employing both capital and labor inputs (See Appendix A for detail equations and solutions). The capital and labor are freely moved between the two sectors. This means prices of the production factors between sectors are the same. The amount of capital and labor used in each sector cannot exceed the total amount available in aggregate.
Market Clearing Conditions
There are four competitive markets in this model: Capital market, Labor market, Good market and Education market. Because the model comprises of heterogeneous individuals, the market clearing equations require aggregating over types of individuals.

For the capital market, an aggregating stock of capital in the economy is simply adding the fraction of individuals with a specific state vector times their asset position $a'$, with the fraction of individuals with a specific state vector times the product of the number of children and bequest value.

For the labor market, the aggregate supply of labor is the sum of the product of the amount of effective labor supplied by a type of individual over all types of individuals.

In the goods market, the aggregate goods is the sum of all goods consumed by a type of individual and his children over all types of individuals plus the aggregate stock of capital that individuals save at the end of the period minus the invested capital that is left from depreciation.

Note that the capital depreciation rate in this model will be set to be one as one period in the model is about 15-20 years. This means the capital is used up during this long period.

In the education market, the aggregate education demand is the sum of the product of the number of children born to a type of parents by the education level which that parent give to the children over the types of all parents. For example, let us consider a fraction of parents who are characterized by an asset level of $a$, an idiosyncratic level of $z$, a number of children ($n_C = 2$), and a university education ($E_c = 3$). This type of individual generates a demand for six units of education.

Solving for value functions
As the utility function used in this research involves altruistic motive, a parent’s value function includes a child’s value function hence the value function of a parent’s becomes an infinite object. As a result, the methods to solve this individual problem involve value function iteration and backward induction. To Solve this problem starts from conjecturing a value function of a child which is the same as young adult’s value function. Using the backward induction method, the optimal decision for the retirement, the old adult, and their value function can be solved by assuming that the retirement saving is zero. Using the optimal old adult value function leads to solutions for the parent’s optimal decisions. This is done by comparing the parents’ utility for
each feasible child education level and parental time level. Using child education level and parental time level and the research algorithm leads to the optimal decision for parental and child consumption, working hours and investment made per child (parental time, education and bequest). Substituting the parent value function into the young adult value function results in the optimal decision and value function of young adult generation. The method of updating the conjectured child value function with the young adult one is repeated until the value function converges.

**Calibration**

This section explains the calibration of parameters: the share of consumption in the utility function, the curvature parameter of the utility function, the discount factor, the capital share parameter in commodity sector, the capital share parameter in education sector, the idiosyncratic shocks, the probabilities of number of children, the parental time parameter and the effective earning profile.

In this model economy, an individual is assumed to live for five 15-year periods. He has the constant relative risk aversion (CRRA) type utility function, $U(c, lt) = [c^\gamma l^{1-\gamma}]^{1-\mu}/(1-\mu)$ with $\gamma$ the share of consumption in the utility function and $\mu$ the curvature parameter. For the share of consumption, we use the value 0.268 as in Chaisrisawatsuk (2004) who used the earning and consumption data from 1990, 1992, 1994, 1996, 1998 and 2000 Thailand Household Socio-Economic and the first order condition for labor supply decision $\gamma/(1-\gamma) = c/w(wt)$ to solve for $\gamma$. Aiyagari (1994) has used three different values for the curvature parameter: 1, 3 and 5. They yielded similar results hence we use the curvature parameter equal to 3. For the discount factor parameter and an altruistic discount factor for the number of children, in the previous studies, the annual discount factor values are 0.91, as in Huggett (1994), 0.924 as in Castaneda et al (2003), 0.934 as in Nishiyama (2000), to 0.96, as in and Aiyagari (1994), Castaneda et al (1998) and Yang (2005). In Chaisrisawatsuk (2004)’s study, the discount factor for a period of 15 years is set to 0.24. The discount factor in this research is set to be 0.24. Based on Knowles’ work (Knowles 1999) the altruistic discount factor parameter is set to be equal to $b(n_c) = n_c^{-\theta}$, where $\theta$ equals to 0.55. This parameter implies that today’s altruistic discount factors with 1-5 children in the model are 1.37, 1, 0.83, 0.73 and 0.66 respectively. Therefore the next period’s altruistic discount factors are $1.37\beta$, $\beta$, $0.83\beta$, $0.73\beta$ and $0.66\beta$ respectively.
The standard Cobb-Douglas production function is used as a functional form. Specifically, the aggregate commodity production function is \( (K_q, N_q) = K_q^\alpha N_q^{1-\alpha} \) and the aggregate education production function is \( Edu(K_e, N_e) = K_e^\lambda N_e^{1-\lambda} \). The parameter \( \alpha \) is the capital share parameter in commodity sector, and \( \lambda \) is capital share parameter in education sector. Tinakorn and Sussangkarn (1998) report capital share data in agriculture, industry, manufacturing and service sector in Thailand from 1980 to 1995. Based on their findings, this research uses the average capital share of agriculture, industry and manufacturing sectors for the commodity sector. This value is 0.60. For the capital share in education sector we average the values of the estimates capital share in the service sector from the 1980 to 1995 in Tinakorn and Sussangkarn (1998), which is 0.61910. 

There are two important types of shock: idiosyncratic shocks and the number of children. The idiosyncratic shocks indicate the probabilities of how well he is doing partially related to his level of education. This type shocks occur at the beginning of each period from 2 to 5. The second type of shock is the probabilities of number of children which occurs at the beginning of the parent period. The values for two random events in the model: idiosyncratic shocks and the number of children are based on the work of Chaisrisawatsuk (2004). The probability transition matrix can be expressed as:

\[
\Pi^e_z = \begin{bmatrix}
\pi_{11}^e & \pi_{12}^e \\
\pi_{21}^e & \pi_{22}^e \\
\end{bmatrix}
\]

(6)

Where \( \pi_{ij}^e \) is a probability of being in state \( i \) for this period entering to state \( j \) for the next period of an agent with education level, \( e \), and for \( i = 1, 2 \) and \( j = 1, 2 \). Let \( e=1 \) represent being in the bad state and \( e=2 \) represent being in the good state. The probability transition matrix is shown below.

<table>
<thead>
<tr>
<th>( \Pi^e_z )</th>
<th>0.62</th>
<th>0.38</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.53</td>
<td>0.47</td>
</tr>
<tr>
<td>( \Pi^e_z )</td>
<td>0.38</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.79</td>
</tr>
<tr>
<td>( \Pi^e_z )</td>
<td>0.27</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.92</td>
</tr>
</tbody>
</table>
To interpret the probability transition matrix, consider the agent with primary education in the bad state, he will stay in the bad state with probability 0.62. In contrast, an individual who starts in a bad state and has an education level of high school will stay in the bad state only 38 percent of the time. Intuitively an agent with primary education, starting in a bad state, will have a higher probability of staying in this state than an agent with an education of high school.

As for a number of children, the unconditional probabilities of numbers of children are as shown.

\[
\Pi_{n_c} = \begin{bmatrix}
\pi_{n_c}^1 & \pi_{n_c}^2 & \pi_{n_c}^3 & \pi_{n_c}^4 & \pi_{n_c}^5 \\
\end{bmatrix} = 
\begin{bmatrix}
0.3924 & 0.3454 & 0.1405 & 0.0717 & 0.05 \\
\end{bmatrix}
\]

where \( \pi_{n_c}^k \) is the probability of having \( k \) children for \( k = 1, 2, ..., 5 \).

The last part of calibration is the earning profile. Based on the work of Chaisrisawatsuk (2004) whose earning profiles are calculated based on age, education level and earning shock, this research is further extended to include parental time effects into the existing earning profile. Parental time spent interacting with children is shown to be a crucial element in the development of the children’s human capital (Casarico (2007)). When the amount of time parents spend on their children is considered from the previous researches, Cardia and Ng (2003) report estimation for the time devoted to childcare by parents a value of 10 hours per week or 9 percent of total time. Cardia and Ng (2003) find that the US that grandparents spend on average almost 9 hours per week or 8 percent of total time looking after grandchildren. Hill and Stafford (1985) report that young households spend between 381 and 813 minutes per week (5.7 percent and 12 percent of total time) on childcare. Hotz and Miller (1988) estimate that the amount of time required to care for a newborn is about 660 hours per year, or 12.69 hours per week (11.3 percent of total time). Leibowitz (1974b) suggests 144.51 minutes per day of an average couple in the survey are spent on physical care of the child, while 131.6 minutes are spent on educational care. These two types of childcare add up to 4.6 hours per day with each spending about 2.3 hours or 14.4 percent of total time. Most of the studies report the time parents spend on their children ranging from 8 to 15 percent of the total time. Nonetheless, the model in this research will cover a wider range of time parents spend on their children from 3 - 24 percent of the total time. Within this range, the parental time parameter is set to 5 levels as shown in the following table:

---

\(^{8}\) Total time is assumed to be 16 hours a day. That is the available time after the sleeping time (assumed to be 8 hours a day) is excluded.
Table 1: Parental time parameter

<table>
<thead>
<tr>
<th>Parental time parameter</th>
<th>Percentage of the total available time of 16 hours</th>
<th>Parental time in total</th>
</tr>
</thead>
<tbody>
<tr>
<td>dt1</td>
<td>3</td>
<td>30 minutes</td>
</tr>
<tr>
<td>dt2</td>
<td>6</td>
<td>1 hour</td>
</tr>
<tr>
<td>dt3</td>
<td>12</td>
<td>2 hours</td>
</tr>
<tr>
<td>dt4</td>
<td>18</td>
<td>3 hours</td>
</tr>
<tr>
<td>dt5</td>
<td>24</td>
<td>4 hours</td>
</tr>
</tbody>
</table>

Given a parental time parameter, the parental time factors can be calculated based on the work of Casarico (2007) as:

$$
\frac{h_c(e, \bar{E}, h, Care, dt2)}{h_c(e, \bar{E}, h, Care, dt1)} = \left[ \frac{dt2}{dt1} \right]^{\sigma_1(1-\omega)}
$$

(7)

This parental time factor is a relative term that compares labor supply efficiency between a group of individuals to the group with the lowest parental time with the same age, education level, and earning shock. The magnitude of the factor depends not only on the differences between parental time but also the value of elasticity of earning with respect to parental time $\sigma_1(1-\omega)$. In Croix and Doepke (2003), the elasticity of earnings with respect to schooling, $\omega$, in actual data goes from 0.4 to 0.8. This research, in line with Casarico (2007), uses 0.6 as a value of $\omega$. For the values of the parameters of the childcare production function, $\sigma_1$ are set to be equal 0.5.

The earning profile can be calculated with the following steps. Firstly the working households are classified by their ages: 16-30 (young); 31-45 (parent); 46-60 (old), and by education level (primary, high school and university). Therefore based on only age and education level, there are 9 groups. For each group, the arithmetic mean of earning is calculated as a reference point. For each individual, if his earning exceeds the reference mean in his group, the individual is in good state of earning shock, otherwise he is considered of being in bad state. By introducing the earning shock, each household group is separated into 2 subgroups (bad and good states). Therefore households can be classified into 18 groups based on ages, education levels and earning shock. In each age-education level-earning shock, average earning is calculated. The labor supply efficiency is computed by divided the average earning of each group by average earning of young primary education with bad state group. Since in the research we classify parental time into 5 levels as in Table 1, for each age-education level-earning shock group, we divide it into 5 groups of parental time level by multiplying the labor supply efficiency by corresponding parental time factors. Eventually the earning profile is classified into 90 groups based on age, education level, parental time and earning shocks (See Appendix B).
Robustness testing
To test the robustness of the model, we compare the results generated from the model with the statistics from the 2000 Household Socio-Economic Survey produced by the National Statistical Office of Thailand. In particular, the results are compared with the Thai data in terms of education distribution, concentration and skewness statistics and the distribution of earning and income. If the values of education distribution, concentration and skewness statistics and the distribution of earning and income from the model are close to the statistics of the Thai data, it can be said that the model is robust and can be used to mimic the Thai economy.

For education distribution, the results generated by the model and the Thai data are quite close. For both primary school and high school, the model slightly under predicts with the percentages of 70.02 and 15.86 respectively, compared to the real values of 72.95 and 16.38 percent. However, it over predicts the fraction of population graduated from university with 14.12 percent compared to the real value of 10.66 percent as shown in Table 2.

Table 2: The Education Distribution for Benchmark Model

<table>
<thead>
<tr>
<th>Education Level</th>
<th>Benchmark Model</th>
<th>2000 Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary School</td>
<td>70.02</td>
<td>72.95</td>
</tr>
<tr>
<td>High School</td>
<td>15.86</td>
<td>16.38</td>
</tr>
<tr>
<td>University</td>
<td>14.12</td>
<td>10.66</td>
</tr>
</tbody>
</table>

The concentration statistics comprise of the Gini coefficients and the ratio of the top 1 percent to lowest 17.8 percent of the population. The Gini coefficients generated by the model for earning and income are 0.56 and 0.53. The values are under predicted, compared with the real values of 0.59 and 0.54 respectively. For the skewness ratio of the top 1 percent to lowest 17.8 percent of the population, the model generates 18.58 for earning and 4.83 for income. Those values are under predicted, compared with the values of the real data of 19.66 and 5.32 respectively.

The skewness statistics consist of the location of mean, skewness coefficient and the ratio of the mean to median. For the location of mean, the model generates 67.80 for earning and 67.49 for income. Both are under predicted when compared with the real values of 68.5 and 70.0 respectively. For ratio of mean to median, the model generates 1.83 for earning compared with the real value of 1.72 while it generates the ratio of 1.64 for income, slightly under predicting the real values of 1.67. For the skewness coefficient, it can be seen that the model under predicts the skewness coefficient of 12.77 for earning and 10.25 for income compared with the real values of 13.61 and 14.5 respectively. Although a few concentration and skewness statistics
from the model departs from those of the real data, most of them suggest that the model can capture the key features that explain the earning and income distribution in Thailand. The concentration and skewness statistics from the model and the real data are summarized in Table 3.

Table 3: Concentration and Skewness Statistics for Benchmark Model

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Earning</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark Model</td>
<td>2000 Data</td>
</tr>
<tr>
<td>Gini</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>Top 1 percent/lowest 17.8 percent skewness</td>
<td>18.58</td>
<td>19.66</td>
</tr>
<tr>
<td>Location of Mean</td>
<td>67.80</td>
<td>68.5</td>
</tr>
<tr>
<td>Skewness Coefficient</td>
<td>12.77</td>
<td>13.61</td>
</tr>
<tr>
<td>Mean/Median</td>
<td>1.83</td>
<td>1.72</td>
</tr>
</tbody>
</table>

In evaluating the model in term of earning and income distribution, the percentage shares of earning and income distribution by quintiles are generated and compare with those from the real data. As it can be seen in table 4, the percentage share of earning and income distribution by quintiles generated by the model are close to those of the real data.

Table 4: Percentage Share of Earning and Income by Quintiles for Benchmark Model

<table>
<thead>
<tr>
<th>Quintiles</th>
<th>Earning</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark Model</td>
<td>2000 Data</td>
</tr>
<tr>
<td>1st</td>
<td>0.97</td>
<td>2.34</td>
</tr>
<tr>
<td>2nd</td>
<td>5.48</td>
<td>4.19</td>
</tr>
<tr>
<td>3rd</td>
<td>11.27</td>
<td>12.46</td>
</tr>
<tr>
<td>4th</td>
<td>24.61</td>
<td>24.74</td>
</tr>
<tr>
<td>5th</td>
<td>57.66</td>
<td>56.26</td>
</tr>
</tbody>
</table>

The results generated from the benchmark model, with parental time to improve children's human capital, can match the shape of education distribution and the shape of earning and income distribution of the real data. Therefore, it can be said that the model with parental time is robust and can be used to mimic the Thai economy. Having made the conclusion on the robustness of the model with parental time, one might question the model’s prediction performance compared with a model without parental time. Should they have the same prediction ability, then it is more efficient to use the model without parental time. In order to confirm that the model with parental time can actually improve the explanatory ability of the previous model, the results generated from both models with and without parental time are compared with the Thai data, once again, in terms of education distribution, concentration and skewness statistics and the distribution of earning and income. If the values of education distribution, concentration and
skewness statistics and the distribution of earning and income from the model with parental time are closer to the statistics of the Thai data than those from the model without parental time, it can be said that the model with parental time can actually improve the explanatory ability of the model without parental time.

When the education distribution generated among the models with and without parental time and the Thai data are compared, while both models under predict the real data, the benchmark model predicts a slightly closer distribution to the real data than the model without parental time. For high school distribution, while the benchmark model slightly under predicts the real data, the model without parental time over predicts the real data. For university education distribution, both models over predict the real values, the model without parental time predicts slightly better. In general the model with parental time predicts closer to the real data as shown in Table 5.

Table 5: Comparison of the Education Distribution among Models

<table>
<thead>
<tr>
<th></th>
<th>Model Without Parental Time</th>
<th>Benchmark Model</th>
<th>2000 Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary School</td>
<td>67.29</td>
<td>70.02</td>
<td>72.95</td>
</tr>
<tr>
<td>High School</td>
<td>21.15</td>
<td>15.86</td>
<td>16.38</td>
</tr>
<tr>
<td>University</td>
<td>11.56</td>
<td>14.12</td>
<td>10.66</td>
</tr>
</tbody>
</table>

For the concentration statistics for both earning and income which are the Gini coefficient and the ratio of the top 1 percent to the lowest 17.8 percent, the benchmark model predicts closer to the real data than the model without parental time. As for the skewness statistics for both earning and income, the result from comparing both models is inconclusive. While the benchmark model predicts closer to the real data than the model without parental time in terms of the location of means of earning, the model without parental time is superior in terms of the location of means of income. For the skewness coefficients of earning the benchmark model predicts closer to the real data than the model without parental time. However, for the skewness coefficients of income, the model without parental time predicts closer than the benchmark model. For the ratio of mean to median, the model without parental time predicts closer to the real data than the benchmark model in terms of earning. However, the benchmark model is superior in terms of income.
Table 6: Comparison of the Concentration and Skewness Statistics among Models

<table>
<thead>
<tr>
<th>Statistic Indicators</th>
<th>Earning</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without Parental Time</td>
<td>Benchmark Model 2000 Data</td>
</tr>
<tr>
<td>Gini</td>
<td>0.51</td>
<td>0.56</td>
</tr>
<tr>
<td>Top 1 percent/lowest 17.8 percent skewness</td>
<td>16.76</td>
<td>18.58</td>
</tr>
<tr>
<td>Location of Mean</td>
<td>66.10</td>
<td>67.80</td>
</tr>
<tr>
<td>Skewness Coefficient</td>
<td>19.73</td>
<td>12.77</td>
</tr>
<tr>
<td>Mean/Median</td>
<td>1.71</td>
<td>1.83</td>
</tr>
</tbody>
</table>

Table 7: Percentage Share of Earning and Income by Quintiles among Models

<table>
<thead>
<tr>
<th>Quintiles</th>
<th>Earning</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without Parental Time</td>
<td>Benchmark Model 2000 Data</td>
</tr>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>1.51</td>
<td>0.97</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>6.61</td>
<td>5.48</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>13.46</td>
<td>11.27</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>28.78</td>
<td>24.61</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
<td>49.64</td>
<td>57.66</td>
</tr>
</tbody>
</table>

Performance comparison of the two models is conducted to explain inequality by examining the percentage share of earning and income distribution by quintiles generated by both models in table 7. It is evident that the overall percentage share of earning distribution by quintiles generated by the benchmark model is better matched the real data than the model without parental time. Interestingly, both models generate similar shapes for income distribution to represent the distribution of real data. Result comparisons in terms of education distribution, concentration and skewness statistics and the distribution of earning and income lead to a conclusion that parental time is another variable that should be included in the model to improve its prediction ability.

**Optimal Parental Time for Thai Economy**

In this section, the benchmark model is used to find the optimal parental time for Thai economy. From development economic studies, a public desired economy should be a result of some combinations of public goals: low earning inequality, low wealth inequality and high total output. To study the optimal parental time for Thai economy, we try to find the solutions based on those assumptions corresponding to three different scenarios. In the first scenario the social goal is to minimize earning inequality (minimize Earning GINI coefficient). In the second scenario, the government is to minimize wealth inequality (minimizing Wealth GINI coefficient). And in the final
scenario the government aims to maximize the total output of the economy. For each scenario, the values of parental time are changed from 3 percent to 33 percent with an increment of 3 percent. The value of parental time that corresponds to the best solution will be the optimal one. In the model, earning is defined as wage payment received from working while wealth is the saving of each individual not including earning. And the total output comprises of total goods consumption, education consumption and aggregate capital supply. Since the model is a closed economy, there is no net export value.

**Parental time to minimize earning inequality**

The elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$ is assumed to be 0.2. The parental time are plotted against corresponding percentage change in earning GINI coefficient in Figure 1. The x-axis represents the values of parental time, presented in terms of ratios of the parental time over total available time, while the y-axis represents the percentage changes in earning GINI coefficient from the earning GINI coefficient of the benchmark model.

**Figure 1: Percentage Change in Earning GINI Coefficient VS Ratio of Parental Time over Available Time**

Enforcing the parental time with children by the government generally improve earning equality from the benchmark model as the percentage changes of earning GINI coefficients are negative for the entire range of parental time. When the parental time is zero the earning GINI is lower than that of the benchmark model by about 7.7 percent. After enforcing the parental time by an average of 3 percent or 30 minutes per day for each child raises earning GINI coefficient by 1.9 percent. When the parental time is at a low level, the earning GINI coefficient grows at an average rate of 0.5 percent for every percent of increase in parental time. As the parental time has been increased up to 9 percent, the percentage changes in earning GINI coefficient reaches about -3 percent. Then an increase in parental time by one percent will raise GINI by only about
0.07 percent. The earning GINI coefficient reaches the maximum level at -2 percent where the parental time is at 15 percent. Further increase in parental time beyond 15 percent results in the decrease of earning GINI coefficient. The percentage change in earning GINI coefficient is at the minimum of -8.1 when the parental time is at beyond 33 percent or about 5 hours and 20 minutes.

The changes in earning GINI coefficient are due not only to the fact that the parental time causes a bigger gap in efficient wages among those who have different education levels and states, but also to how different groups adjust their working time given higher efficient wages. Although an increase in efficient wages makes individuals adjust their working hours, the low earners and high earners adjust working hours differently. When the parental time is low, an increase in parental time causes the low earning groups to reduce their working time comparatively more than the high earning groups. Hence earning gaps grow at this stage. When parental time is high the increase in efficient wages again affects the lower earners more than the higher ones in term of working hours. However, the effect is in the opposite direction. By raising the already high parental time, low earners in all generation will work harder, narrowing earning gaps.

**Parental time to minimize wealth inequality**

With the elasticity of earning with respect to parental time $\sigma_1(1 - \omega)$ is 0.2, the percentage changes in wealth GINI coefficient are plotted against parental time in Figure 2.

**Figure 2: Percentage Change in Wealth GINI Coefficient VS Ratio of Parental Time over Available time**
At the start, enforcing the parental time by an average 3 percent or 30 minutes per day for each child actually decreases wealth GINI coefficient to -5.2 percent. A further 3 percent increase in parental time will decrease the wealth GINI coefficient for another 5.3 percent. Then the wealth GINI coefficient decreases at a slower rate of 0.34 percent for every percent of increase in parental time. As the parental time has been increased up to 15 percent the percentage change in wealth GINI coefficient reaches the minimum value of about -13.5 percent. Then from the minimum point an increase in parental time by a percent will raise the wealth GINI coefficient by about 0.37 percent. The percentage change in wealth GINI coefficient reaches -4.94 percent as the parental time is at 33 percent.

When the parental time is small, increasing it causes individuals with primary education to increase their savings at the higher rate than others. On the other hand, high earners can afford not to increase their saving as much for smoothing consumption as their wages are higher for future periods. When the parental time is between 0 percent and 6 percent the wealth GINI coefficient decreases. When the parental time reaches about 6-9 percent, further increase in parental time does not affect the saving of low earners as much as it does at the beginning. The wealth GINI coefficient still decreases but at a lower rate. When the parental time is between 9-15 percent all individuals keep saving about the same amount. And during this area the gap between the richest and the poorest in term of saving is the closest. Increasing parental time of greater than 15 percent will result in the low earners to gradually decrease their saving, making the wealth GINI coefficient to increase.

**Parental time to maximize total output**

The elasticity of earning with respect to parental time, $\sigma_1(1 - \omega)$, is assumed to be 0.2. In Figure 3, the parental time are plotted against corresponding percentage change in total outputs. On the x-axis, the values of parental time is presented in term of ratios of the parental time over total available time while on the y-axis, the percentages of total outputs are calculated as percentage change in total output from the total output of the benchmark model.

When the government enforces the parental time from start by an average of 3 percent or 30 minutes per day for each child, the percentage change in total output rises by around 2.8 percent. As the parental time increases for another 3 percent from 3 - 6 percent, the percentage change in total output sharply rises by another 7 percent. The rate of increase in the percentage change in total output from a percentage increase in parental time at this stage is at the highest. After the parental time has reached 6 percent, further increase in parental time cause the percentage change in total output to raise at a decreasing rate. It is noted that the percentage change in total output reaches the maximum value of 14.67 percent when the parental time is
at 18 percent or about 3 hours per day. Further increase in parental time beyond 18 percent leads to the percentage change in total output to decrease although it is still positive compared with the total output of the benchmark model.

**Figure 3: Percentage Change in Total Output VS Ratio of Parental Time over Available Time**

![Graph showing percentage change in total output vs ratio of parental time over available time.](image)

Increasing parental time results in increases in the human capital for the entire population. These increases in human capital are reflected through higher efficient wages for each level of education. With higher efficient wages, individuals can earn more from working for the same amount of working time. As for the economy as a whole, an increase in parental time means an increase in the aggregate labor productivity. Given the same amount of aggregate capital, the total output of the economy can be more produced. However, the increase in parental time also comes with costs. Parents will have to reallocate the time left available from taking care of children to leisure and work. In general, this means that parents will have less working time which in turn has a negative effect on the labor supply. Although enforcing more parental time from parents increase efficient wages, its cost is fewer working hours for parents. So for labor supply in aggregate, as the parental time increases labor supply increases at the beginning. For further increasing in parental time, the cost of loosing parent labor supply grows such that it slows down the increase in percentage change in labor supply. Finally at the high level of parental time the percentage change in labor supply decreases. To sum up, when the parental time is at a low level, the benefits of enforcing more parental time exceed the cost of reducing working time. Hence the total output along with wages and saving increases. Once the level of parental time is already high, and its further increase causes the costs such as reduction of parents’ working time and total saving to exceed the benefit of lifting efficient wages. Hence, the total output is maximized at the parental time of 18 percent.
In order to find the optimal parental time for Thai economy, we explore the robustness of the result by altering the values of elasticity of earning with respect to parental time. In this section the values of elasticity of earning with respect to parental time take the values of 0.1, 0.15, 0.2, 0.25 and 0.3 respectively. Then for each scenario, the optimal solutions are calculated against parental time with different values of elasticity of earning. If the results are robust each value of elasticity of earning should give the same optimal solution. Table 8 summarizes the optimal parental time for different value of elasticity of earning with respect to parental time. We find that for each scenario the optimal parental times are not exactly at the same level but each optimal parental time corresponding to different values of elasticity of earning with respect to parental time are very close.

Table 8: Summary of Optimal Parental Time for Thai Economy

<table>
<thead>
<tr>
<th>(\sigma_1(1 - \omega))</th>
<th>Percent Parental time when Earning GINI at Maximum</th>
<th>Percent Parental time for Minimum Earning GINI Coefficient</th>
<th>Percent Parental time for Minimum Wealth GINI Coefficient</th>
<th>Percent Parental time for Maximum Total Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>12 (2 hours)</td>
<td>33 (5 hours 20 minutes)</td>
<td>12 (2 hours)</td>
<td>12 (2 hours)</td>
</tr>
<tr>
<td>0.15</td>
<td>15 (2 hours 24 Minutes)</td>
<td>33 (5 hours 20 minutes)</td>
<td>12 (2 hours)</td>
<td>18 (3 hours)</td>
</tr>
<tr>
<td>0.2</td>
<td>15 (2 hours 24 Minutes)</td>
<td>33 (5 hours 20 minutes)</td>
<td>15 (2 hours 30 minutes)</td>
<td>18 (3 hours)</td>
</tr>
<tr>
<td>0.25</td>
<td>18 (3 hours)</td>
<td>33 (5 hours 20 minutes)</td>
<td>12 (2 hours)</td>
<td>21 (3 hours 20 minutes)</td>
</tr>
<tr>
<td>0.3</td>
<td>18 (3 hours)</td>
<td>33 (5 hours 20 minutes)</td>
<td>15 (2 hours 30 minutes)</td>
<td>21 (3 hours 20 minutes)</td>
</tr>
</tbody>
</table>

Setting parental time allocation according to one specific social goal is a straight forward step. However, choosing the best solution seems to be a difficult task, given an unclear social goal as shown in Table 8. For example, with the elasticity of earning with respect to parental time equal to 0.2, setting the solution for parental time to 18 percent to achieve the maximum total output will come with a cost of widening the earning inequality. On the other hand, minimizing earning inequality by either keeping the parental time allocation at minimum or setting it to the maximum will not improve the situation in total output or on wealth inequality. However, when the social goal is unclear, it seems reasonable to set the target on maximizing the total output which automatically improves wealth equality bearing the cost of earning inequality being around the widest among the entire range of parental time. Although enforcing more parental time may widen the human capital gap compared with when enforcing less parental time, the earning equality has been improved from the benchmark model.
Having known the current situation about parental time and having set a social target on one of the desired parental time, what policies should be implicated to move the parental time from the current situation to the specific target? There are a few directions to improve the current human capital based on parental time: to improve quantity and quality of parental time. Firstly, to improve on the quantity of parental time, a government can introduce an incentive that parents value high enough to give up working time and leisure time for taking care of children such as tax-related incentives. With less working time parents can allocate more time taking care of children, increasing parental time. A subsidy policy can be implemented for parent generation as well. This subsidy should work as an earning insurance such that parents with high earnings can work less and take care of their children more.

Secondly as for improving the quality of parental time, the elasticity of earning with respect to parental time, $\sigma_1(1 - \omega)$, can be viewed as the quality component of parental time. When the parental time is relatively less productive, the elasticity of earning with respect to parental time may be at the lowest level of 0.1 in this study. The quality of parental time is not so good. For a parent with very high value of elasticity of earning with respect to parental time, 0.3 in this study, his quality of parental time is high. Given the same parental time, a parent with higher elasticity of earning with respect to parental time or higher quality of parental time, should raise children with higher earning capability. Taking an example from Figure 6, in order to raise the total output up to around 15 percent compared to the benchmark model, the parents with higher quality parental time ($\sigma_1(1 - \omega)=3$) require only 5 percents of the total time with their children while the parents with lower quality parental time ($\sigma_1(1 - \omega)=1.5$) require 11 percents of their total time. This implies that keeping the quantity of parental time fixed, the total output of the economy can be improved by improving the quality of those parental time. In this sense a policy recommendation for the government would be to improve the quality of parental time for those with very low level of elasticity to the standard level of society. For example, the government can build a better environment for children to be raised in or enhance parents’ awareness on the benefit of parental time.

4. Conclusion
The first objective of this study is to construct an economic model with intergenerational transfer within family that includes bequest, education and parental time, in order to mimic the earning and income inequality of Thailand. The second objective is to determine the optimal parental time for the model economy to achieve three social goals; minimizing earning and wealth inequality, and maximizing total output.
A five period overlapping generation model is constructed and differs from a general heterogeneous overlapping generation model in many ways. While recent studies focus on both unplanned and planed bequests (money and education) as intergenerational links, a parental time variable is added into the economic model. Parental time variable is another channel besides education that helps children develop their human capital which in turn increases efficient wage and wealth in the future. The calibrated parameters from previous studies are used and the results are compared with some statistics, in particular, education distribution and the earning and income distribution, generated by a previous study from the 2000 Household Socio-Economic Survey.

In general, the model generates the education distribution and the earning and income distribution reasonably close to those of the real data. Then the parental time link in the model is disabled, keeping all parameters with same values. The finding is that the model without parental time is not as good as the benchmark model in term of mimicking the education distribution and earning distribution. Therefore, it is concluded that parental time link is important and that the benchmark model can be used as a reference economy for any future analysis.

In order to find the optimal parental time for Thai economy, different values of parental time in the model are altered for three different scenarios.: minimize earning GINI coefficient and wealth GINI coefficient and maximizing the total output. For each scenario, the model generates various results in different environments using different values of elasticity of earning with respect to parents’ parental time. To minimize wealth inequality the parental time should be set at 12-15 percent, and to maximize the total output, the average parental time should be set at 12-21 percent of available time. While increasing the parental time will be beneficial in improving output and wealth equality, it comes with a cost of widening the earning gaps.

Suggestions are presented here to improve the current situation on parental time: to improve on quantity and quality of parental time. To improve on the quantity of parental time, a government can introduce a policy to create an incentive for parents to give up working and leisure time for taking care of children. As far as the quality of parental time is concerned, a government can help improving the quality of parental time by creating better environment for children or enhancing parents’ awareness on the benefit of the parental time.
References


Appendix A

The household utility function is of the constant relative risk aversion (CRRA) type:

\[ U(c, \ell t) = \frac{[\gamma \ell t^{1-\gamma}]^{1-\mu}}{1-\mu} \quad A-1 \]

where \( \gamma \) is the share of consumption in the utility.
\( \mu \) is the curvature parameter.
\( \ell \) is the leisure time.

The optimization problem for the individual

The second period:

\[ v(j=2, a=beq, \ell t, e, z, n_c=0) = \max_{\{c, \ell t, a\} \in [0, \infty]} \left\{ u(c, (1 - \ell t)) + \beta E[v(j + 1, a', \ell t, e, z', n_c | z)] \right\} \quad A-2 \]

subject to

\[ c + a' \leq wn(j, \ell t, e, \ell t, z) + (1 + r) a \quad A-3 \]

Where \( \beta \) is the discount factor.
\( E \) is the expectation operator.

The solutions derived from the optimal condition are:

\[ c(j = 2) = \gamma[w - a' + (1 + r) beq] \quad A-4 \]
\[ \ell t(j = 2) = 1 - \left[ \frac{(1-\gamma)c}{\gamma w} \right] \quad A-5 \]
\[ v(j = 2) = \frac{[\gamma \ell t^{1-\gamma}]^{1-\mu}}{1-\mu} + \beta E v(j) \quad A-6 \]

The Third period:

\[ \max_{\{c, \ell t, a', c', \ell t, e, c, beq\}} \left\{ u(c, (1 - \ell t - n_c \ell t)) + b(n_c) n_c u(c^k) + \beta b(n_c) n_c E[v^c(j = 2, a = beq, \ell t, e, c, z', n_c = 0 | z)] + \beta E [v(j + 1, a', \ell t, e, z', n_c = 0 | z)] \right\} \quad A-7 \]
subjected to
\[ c + n_c c^k + a' + n_c TC(e_c) + n_c beq \leq wn(j, dt, e, wt, z) + (1 + r)a \quad A-8 \]

The solutions derived from the optimal condition are:

\[ c(j = 3) = \frac{[w-a'-n_c TC(e_c)-n_c beq+(1+r)a-wn_c dt_c]}{1+\frac{1-\gamma}{\gamma}} \frac{1}{[\gamma w]} \frac{1}{\gamma} \frac{1}{\mu} n_c^\epsilon_2 \quad A-9 \]

\[ c^k = \left[ \frac{1-\gamma}{\gamma w} \right]^{\epsilon_1} \cdot \gamma^{-\frac{1}{\mu}} \cdot n_c^{\epsilon_3} \cdot c(j = 3) \quad A-10 \]

\[ wt(j = 3) = 1 - n_c dt_c - \left[ \frac{(1-\gamma)c}{\gamma w} \right] \quad A-11 \]

\[ v(j = 3) = \frac{c^y(1-wt-n_c dt_c)^{1-\gamma}}{1-\mu} + n_c^{\epsilon_1} \cdot \left( c^k \right)^{1-\mu} + \beta n_c^{\epsilon_1} \cdot Ev^c + \beta Ev(j =) \quad A-12 \]

Where

\[ \epsilon_1 = (1 - \gamma) - \left[ \frac{1-\gamma}{\mu} \right] \quad A-13 \]

\[ \epsilon_3 = \frac{\epsilon_1 - 1}{\mu} \quad A-14 \]

\[ \epsilon_2 = \epsilon_3 + 1 \quad A-15 \]

\[ \epsilon_1 = 1 \cdot \theta \]

The forth period:

\[ v(j=4, a, dt, e, z, n_c=0) \quad A-16 \]

subjected to

\[ c + a' \leq wn(j, dt, e, wt, z) + (1 + r)a \quad A-17 \]
The solutions derived from the optimal condition are:

\[ c(j = 4) = \gamma [w - a' + (1 + r)a] \quad \text{A-18} \]

\[ wt(j = 4) = 1 - \left[ \frac{(1-\gamma)c}{\gamma w} \right] \quad \text{A-19} \]

\[ v(j = 4) = \frac{e^{r(1-wt)}(1-\gamma)^{1-\mu}}{1-\mu} + \beta E v(j = 5) \quad \text{A-20} \]

The fifth period:

\[ v(j=5, a, dt, e, z=0, n_c=0) = \max_{\{c\}} \{u(c)\} \quad \text{A-21} \]

subject to

\[ c = (1 + r)a \quad \text{A-22} \]

The solutions derived from the optimal condition are:

\[ c(j = 5) = (1 + r)a \quad \text{A-23} \]

\[ wt(j = 5) = 0 \quad \text{A-24} \]

\[ v(j = 5) = \frac{e^{1-\mu}}{1-\mu} \quad \text{A-25} \]

Problems of the Firms

Good market.

\[ Y = F^g(K_g, N_g) = K_g^{\alpha}N_g^{1-\alpha} \quad \text{A-26} \]

Education market

\[ E = F^e(K_e, N_e) = K_e^{\lambda}N_e^{1-\lambda} \quad \text{A-27} \]

Where

- \( Y \) represents the amount of commodity goods that firm produces
- \( E \) represents the quantity of education
- \( K_g \) is the aggregate capital used in commodity goods sector
- \( K_e \) is the aggregate capital used in education sector
- \( N_g \) is the aggregate labor employed in commodity goods sector
- \( N_e \) is the aggregate labor employed in education sector
- \( \alpha \) is the capital share parameter in commodity goods sector
- \( \lambda \) is the capital share parameter in education sector
\( p_g \) is the producer output price which is normalized to unity
\( p_e \) is the education price
\( r_g \) is the real interest rate in the commodity goods market
\( r_e \) is the real interest rate in the education market
\( w_g \) is the real wage rate in the commodity goods market
\( w_e \) is the real wage rate in the education market

The firm’s profit maximization problems are as follows:

\[
\max_{[K,q,N_q]} p_g K_g^\alpha N_g^{1-\alpha} - r_g K_g - w_g N_g
\]  \tag{A-28}

The capital and labor are freely moved between the two sectors. This means prices of the production factors between sectors are the same. That is:

\[
\begin{align*}
    r &= r_g = r_e \quad \text{(A-29)} \\
    w &= w_g = w_e \quad \text{(A-30)}
\end{align*}
\]

The amount of capital and labor used in each sector cannot exceed the total amount available in aggregate. This restriction can be expressed as:

\[
\begin{align*}
    K &= K_g + K_e \quad \text{(A-31)} \\
    N &= N_g + N_e \quad \text{(A-32)}
\end{align*}
\]
## Appendix B

### The Earning Profile

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