Research Article

THE CONSTRUCTION AND VALIDATION OF THE COGNITIVE MODEL OF FORCE AND MOTION FOR A DIAGNOSIS OF MISCONCEPTIONS

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Abstract

The cognitively diagnostic assessment (CDA) gives importance to learners' thinking processes in doing a test. Its aim is to provide detailed feedback to learners on their mastery of predetermined attributes to improve their learning. It is appropriate to apply the CDA to construct diagnostic tests to uncover misconceptions about force and motion. We classified 27 synthesized misconceptions about force and motion into six categories: 1) resultant force, 2) Newton's first law of motion, 3) Newton's second law of motion, 4) Newton's third law of motion, 5) frictional force, and 6) gravitational force. Then, we determined the hierarchical relationships among categories to construct the cognitive model of force and motion, which was validated by seven experts on the subject matter. The preliminary results in this study are the basis for the construction of diagnostic tests integrated with the CDA. The guidelines for constructing the tests to diagnose misconceptions about force and motion are discussed in the discussions and suggestions section.

Keywords: Cognitive Model, Misconceptions, Force and Motion, Cognitively Diagnostic Assessment

Introduction

Mechanics is essential to learning physics, and the force and motion concept is at the heart of the study of mechanics. Moreover, the concept of force and motion is a prerequisite for understanding other physics concepts, such as waves, heat, and electricity (Tomara et al., 2017). If learners possess misconceptions about force and motion, their grasp of mechanics will be meaningless. They will not be able to apply the concept of force and motion to other complicated concepts either. Eventually they will fail to achieve the academic goals of learning physics. It is essential that learners correctly understand the concept of force and motion to achieve proficiency in physics (Gurel et al., 2015; Hestenes et al., 1992).

Learners have been found to have misconceptions about force and motion (Handhika et al., 2016; Kurniawan, 2018). It is thus necessary to develop diagnostic tests to assess learners' misconceptions. Data collected from the test will be used to design course materials that would correct these misconceptions. The instruments widely used to diagnose misconceptions about force and motion are the Force Concept Inventory (FCI) and the Force and Motion Conceptual Evaluation (FMCE) (Stewart et al., 2018). These instruments are based on the classical test theory (CTT) and the item response theory (IRT), both of which overlook learners' thinking processes despite the fact that their thinking processes are vital to diagnostic tests (Nichols, 1994; Rupp & Templin, 2008).

Designing diagnostic tests should take into account learners' thinking processes and knowledge structures in order to gather diagnostic information to provide detailed feedback to learners about their mastery of the predetermined set of attributes or skills. This is the key characteristic of the cognitively diagnostic assessment (CDA). The CDA is an approach that integrates cognitive psychology into psychometric modeling. It underscores a formative assessment, which provides detailed feedback about the weaknesses and strengths regarding learners' assessed attributes. It is aimed at helping learners improve and develop their personal learning more than preparing them for a competitive examination or ranking learners. The thinking processes presented as a cognitive model is identified by using information from cognitive theories and psychology theories to construct a Q-matrix, a table that indicates the attributes required to get the answer correct for each item. It also functions as a guideline for creating a test and analyzes answers using the cognitive diagnostic models (CDMs) (Javidanmehr & Sarab, 2017; Nichols, 1994; Rupp & Templin, 2008).

Not only does the CDA provide detailed feedback to individual learners about their weaknesses and strengths, but it also provides information to instructors to plan learning materials and assess the efficacy of learning management (Hung & Huang, 2019). As a result, interest in the CDA increases nowadays (Ma & de la Torre, 2019).

CDAs are divided into five major steps: 1) defining the purpose of the assessment, 2) specifying and validating attribute specifications, 3) constructing and validating the Q-matrix, 4) selecting CDMs for a data analysis, and 5) reporting assessment results. The first step is clearly defining the purpose of the assessment and describing the attributes to be assessed. The next step is identifying a set of attributes assessed in a test and specifying the hierarchical relationships among attributes, known as the cognitive model. Then, the cognitive model is validated. There are three methods to validate the cognitive model: (a) think-aloud protocols, (b) eye-tracking studies, and (c) expert panels. In the third step, a Q-matrix is developed to identify the relationships between the target attributes and individual items based on information obtained from the cognitive model. Then, the Q-matrix is validated. There are three methods to validate the Q-matrix: (a) think-aloud protocols, (b) expert panels, and (c) empirical data analysis based on the CDM. Developing the cognitive model and the Q-matrix is a recurring process that needs repeated revisions until satisfactory results are obtained. The completed Q-matrix is used as a guideline for the development of test items. In the fourth step, learners' answers to the test's items are analyzed. The chosen CDM and the completed Q-matrix are used to estimate each learner's profile of their mastery of attributes and item parameters. The final step is reporting the profiles of mastery of attributes to each learner according to the data analysis. The score reports could be customized for each learner by including information about the learner's mastery status, strengths, weaknesses, and recommended remedial programs. The score reports are presented in graphics and written accounts (DiBello et al., 2007; Javidanmehr & Sarab, 2017; Lee & Sawaki, 2009).

Presently, CDMs are designed to diagnose skills, misconceptions, or both. However, there are a few CDMs available to diagnose misconceptions (Kuo et al., 2018). They have been developed over the past decade. Examples of CDMs for identification misconceptions are the bug deterministic inputs, noisy or gate model (Bug-DINO; Kuo et al., 2016), and the scaling individuals and classifying misconceptions model (SICM; Bradshaw & Templin, 2014).

Demand for the use of CDMs to diagnose misconceptions has increased due to the need to provide informative diagnostic feedback to improve learners' learning. However, analyzing data regarding misconceptions about force and motion is based solely on the classical test theory (CTT) and the item response theory (IRT).

Nowadays, there are no diagnostic tests that apply the CDA to study misconceptions about force and motion. Moreover, there still lacks the cognitive model of force and motion, the key component of the Q-matrix construction, which further poses a major challenge to creating new diagnostic tests that adopt the CDA to diagnose misconceptions about force and motion (Lee & Sawaki, 2009). We, therefore, synthesized and put misconceptions about force and motion into groups. Moreover, we constructed the cognitive model of force and motion determined by the hierarchical relationships among these categories and turned to experts on the subject matter for validation.

Literature Review

1. Misconceptions about force and motion

Misconceptions refer to ideas differing from accepted scientific explanations that impede learning (Bayraktar, 2009; Gurel et al., 2015). Table 1 shows 27 common misconceptions about force and motion possessed by learners in secondary and higher education. The data were acquired through literature reviews (Alonzo & Steedle, 2009; Atasoy et al., 2011; Bayraktar, 2009; Eryilmaz, 2002; Halloun & Hestenes, 1985; Hestenes et al., 1992; Liu & Fang, 2016; Martin-Blas et al., 2010; Narjaikaew, 2013; Poutot & Blandin, 2015; Tomara et al., 2017).

Table 1 Misconceptions about force and motion

Groups	Misconceptions	Percentage of misunderstanders
1. Resultant force	1.1 An object moves in the direction of the greater force.	54% - 63%
	1.2 An object changes its direction in the direction of the last force.	18% - 39%
2. Newton's first law of motion	2.1 An object stores an applied force into an impetus to keep the object going after the force is worn out.	40% - 64%
	2.2 An impetus keeps objects moving.	44% - 65%
	2.3 A trajectory of an object depends on an impressed impetus.	6% - 38%
3. Newton's second law of motion	3.1 If there is no motion, there is no force acting on an object.	3% - 10%
	3.2 A moving object stops when the force is stopped.	20% - 47%
	3.3 If there is motion, there is a force acting on an object in its direction of motion.	26% - 46%
	3.4 If there is a force acting on an object at rest, the object will move.	11% - 18%
	3.5 When an object is moving, there is a force in the direction of its motion.	12% - 24%
	3.6 There is a linear relationship between force and velocity. In other words, a constant velocity results from a constant force.	13% - 66%

Groups	Misconceptions	Percentage of misunderstanders
	3.7 An object that moves with a constant acceleration requires a constantly changing force.	23% - 40%
	3.8 Forces are caused by living or moving things.	17% - 40%
	3.9 Forces can only be caused by something touching an object.	37% - 46%
4. Newton's third law of motion	4.1 An action-reaction pair of force acts on the same object.	60% - 80%
	4.2 According to applied forces between two objects, the greater mass exerts the greater force.	43% - 51%
	4.3 According to applied forces between two objects, the bigger object exerts the greater force.	72% - 84%
	4.4 According to applied forces between two objects, the most active object exerts the greater force.	28% - 83%
	4.5 When an object moves into an obstacle, the obstacle redirects or stops motion but it cannot be the agent of an applied force.	29% - 64%
5. Frictional force	5.1 Frictional force acts on an object when it moves.	25% - 68%
	5.2 Frictional force always acts opposite to the direction of motion.	25%
	5.3 Static frictional force is minimum when an object begins to move.	23%
	5.4 Static frictional force is constant and equals to a coefficient of static friction multiplied by a normal force.	39% - 40%
6. Gravitational force	6.1 For free fall, a heavier weight causes a bigger acceleration. In other words, heavier objects fall faster.	12% - 54%
	6.2 There is the gravitational force acting on an object when it is only on the earth.	13% - 70%
	6.3 The gravitational force has constant value and is the same everywhere.	4% - 8%
	6.4 The gravitational force does not act until an impetus wears down.	19% - 37%

2. The cognitive model

A cognitive model or an attribute specification presents learners' thinking processes involved in doing a test. The construction of a cognitive model requires the consideration of the hierarchical relationships among attributes. The attribute hierarchies are specifications of the relationships among attributes that suggest that mastering an attribute requires mastering others first. There are five types of attribute hierarchy as follows (Rupp et al., 2010; Tu et al., 2019):

A linear attribute hierarchy presents a hierarchy in which all attributes are sequentially ordered in a single chain. It implies that if the first attribute is not present, all following attributes will not be present.

A convergent attribute hierarchy presents a hierarchy in which a single attribute has multiple prerequisite attributes in a single chain. In other words, the multiple parent attributes converge in the same common child attribute.

A divergent attribute hierarchy presents a hierarchy in which multiple distinct branches originate from the same attribute. In other words, different branches of the child attribute diverge from the common parent attribute.

An unstructured attribute hierarchy presents a hierarchy in which one attribute is a prerequisite for other distinct attributes. In other words, the common child attributes that are unrelated to each other are linked to the same common parent attribute.

A mixed structure presents a hierarchy in which there are more than one set of attributes, and each set of them has an independent structure. For example, there are two independent sets of attributes where the first set is a linear structure, while the second set is a convergent structure, as shown in Figure 1.

It is concluded that attribute hierarchies can be divided into five types, i.e., 1) a linear attribute hierarchy, 2) a convergent attribute hierarchy, 3) a divergent attribute hierarchy, 4) an unstructured attribute hierarchy, and 5) a mixed structure. The first four types have only one set of attributes, while a mixed structure contains more than one set of attributes. There is a single attribute chain in a linear and convergent attribute hierarchy, but there are multiple attribute chains in a divergent attribute hierarchy. However, all attributes are sequentially ordered in a linear attribute hierarchy, while those are not needed to be sequentially ordered in a convergent attribute hierarchy. In an unstructured attribute hierarchy, one attribute is a prerequisite of several distinct attributes.

Apart from the five types of attribute hierarchies, there is another structure called an independent structure, which stipulates that the mastery of an attribute does not require the mastery of other attributes. Figure 1 shows all five types of attribute hierarchy and an additional independent structure.

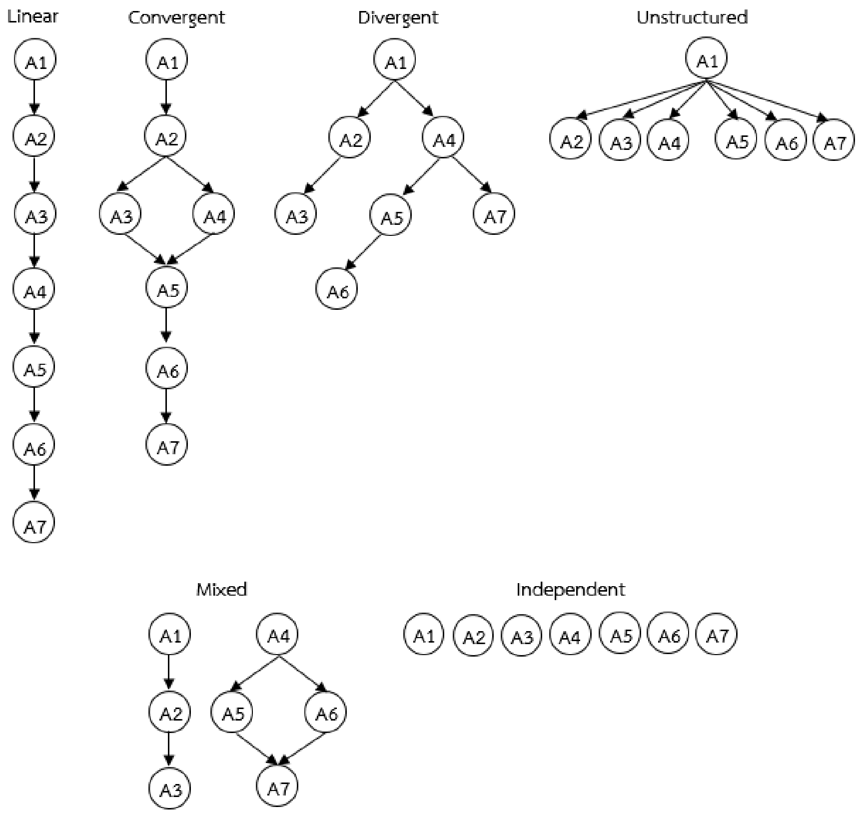

Figure 1 Types of attribute hierarchy

The cognitive model has four characteristics: (1) the model contains attributes that are specified at a fine-grained size, (2) the attributes must be measurable, (3) the attributes must be instructionally relevant to a broad group of educational stakeholders, and (4) the model reflects a hierarchy of ordered attributes within a domain (Gierl et al., 2010).

There are three methods to validate cognitive models: (1) think-aloud protocols, (2) eye-tracking studies, and (3) expert panels. The first method, think-aloud protocols or verbal protocols, requires learners to think aloud while solving an item (concurrent verbal protocols) or after they complete an item (retrospective verbal protocols) to confirm or disconfirm the hierarchical relationships among attributes in the cognitive model. This method is the most important one to validate cognitive models. The second method, eye-tracking studies, is based on the same logic as think-aloud protocols. It utilizes imaging techniques, in which the gaze of each individual learner is tracked as they are taking each test item. However, this method can be used to great effect when used as complementary data to the qualitative data collected as part of the think-aloud protocols and to the quantitative data in which the time taken to answer a test item is recorded. The last method, the cognitive model is validated by asking the experts who are asked to describe the cognitive response process when learners solve the problems based on prior research and the experience of assessments in a particular domain (Gorin, 2007; Rupp et al., 2010).

Research Methodology

We reviewed, synthesized, and grouped common misconceptions about force and motion possessed by learners in secondary and higher education. In categorizing these misconceptions, we followed the taxonomy of misconceptions proposed by Hestenes et al. (1992) and the characteristics of the cognitive model proposed by Gierl et al. (2010). Then, we determined the hierarchical relationships among categories or attributes to construct a cognitive model of force and motion. Next, details of the misconceptions, the misconceptions categories, and the cognitive model were validated by seven experts on physics education. The criterion for selecting the experts stipulates they must have at least five years of teaching experience. Semi-structured interviews were conducted, and an interview record form was used for the interviews. Interview issues included obviousness of each misconception, and appropriateness of misconceptions categories and the cognitive model. Each interview was recorded with the experts' permission. The recorded interviews were transcribed to analyze the data. For data analysis, a content analysis was employed. Finally, we revised the materials according to the experts' suggestions.

Research Findings

1. **Misconceptions about force and motion categories**

We grouped 27 synthesized misconceptions about force and motion into six categories: 1) resultant force, 2) Newton's first law of motion, 3) Newton's second law of motion, 4) Newton's third law of motion, 5) frictional force, and 6) gravitational force. Table 1 lists these misconceptions in each group. However, we classified the impetus idea into the category of Newton's first law of motion because this idea contradicts Newton's first law of motion, which demonstrates learners' misunderstanding of this law. That is, learners believe that a resultant force is required to keep an object moving at a constant velocity. Impetus is an imaginative force for learners. They believe that the impetus is an inanimate motive power or an intrinsic force that comes from an applied force and keeps objects moving. Impetus can be gained or lost in a variety of ways, according to each learner's beliefs (Hestenes et al., 1992).

2. Cognitive model of force and motion

The cognitive model of force and motion illustrates hierarchical relationships among attributes or categories of misconceptions about force and motion. In other words, the model describes that having a misconception about an attribute will lead to having misconceptions about other attributes. The model is shown in Figure 2 and can be explained as follows.

Concepts of resultant force and Newton's first law of motion are prerequisites for Newton's second law of motion. Thus, if learners possess misconceptions about resultant force and Newton's first law of motion, they will possess misconceptions about Newton's second law of motion. That is, learners who incorrectly understand that an object moves in the direction of the greater force or in the direction of the last force – a misconception about resultant force – will incorrectly understand that there is a force acting on an object in its direction of motion – a misconception about Newton's second law of motion. Moreover, understanding the impetus idea as an object storing an applied force into an impetus to keep the object going after the force is worn out is a misconception about Newton's first law of motion. It reflects learners' incorrect understanding of the fundamental force knowledge that leads to an incorrect understanding that every motion has a cause called the causal principle of motion. Consequently, learners will incorrectly understand the concept of Newton's second law of motion, thinking there is a force in the direction of its motion when an object is moving and that there is no force acting on an object at rest. Additionally, learners who believe in the causal principle of motion will possess the key misconception about frictional force (i.e., frictional force acts on an object when it moves). That is, if they incorrectly understand that there is no force acting on an object at rest, then they will incorrectly understand that there is no frictional force acting on an object on a frictional medium when it is acted on but is not moving. It explains why learners who misunderstand the concept of Newton's second law of motion will misunderstand the concept of frictional force.

Not only do learners who possess misconceptions about Newton's second law of motion have misconceptions about frictional force, they also possess misconceptions about gravitational force. Because Newton's second law of motion involves resultant force, mass, and acceleration, this law is fundamental to learning gravitational force. Thus, if learners possess misconceptions about Newton's second law of motion, then they will possess misconceptions about gravitational force. That is, learners who incorrectly understand about Newton's second law of motion that an object that moves with a constant acceleration requires a constantly increasing force will incorrectly understand that heavier objects fall faster – a misconception about gravitational force. Furthermore, learners who incorrectly understand that forces can only be caused by something that touches an object – a misconception about Newton's second law of motion – will incorrectly understand that there is a gravitational force acting on an object when it is only on the earth – a misconception about gravitational force. Nonetheless, Newton's third law of motion has no hierarchical relationship with other attributes.

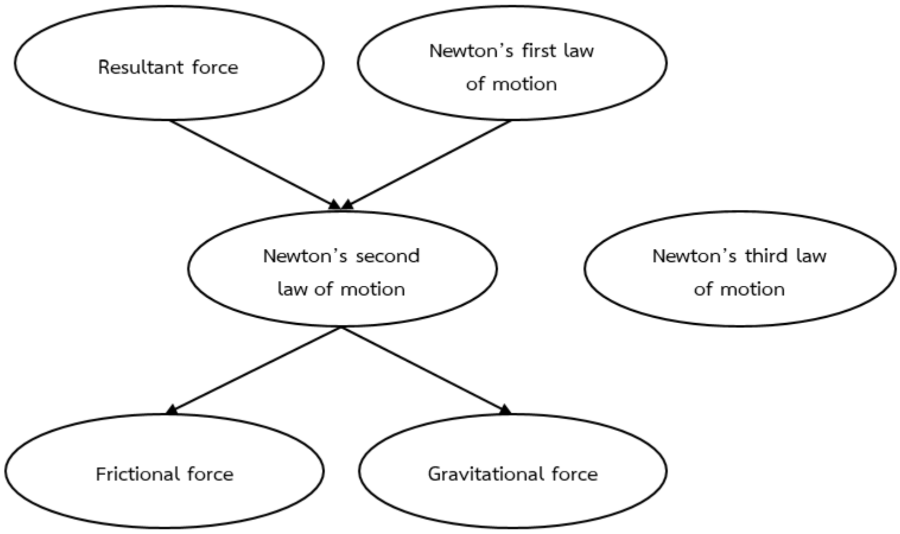

Figure 2 The cognitive model of force and motion

Discussions and Suggestions

The findings can be summarized as follows: (1) misconceptions about force and motion can be grouped into six categories: (a) resultant force, (b) Newton's first law of motion, (c) Newton's second law of motion, (d) Newton's third law of motion, (e) frictional force, and (f) gravitational force. And, (2) the cognitive model of force and motion illustrates that learners who possess misconceptions about resultant force and Newton's first law of motion will possess misconceptions about Newton's second law of motion. Then, they will possess misconceptions about frictional force and gravitational force. Moreover, Newton's third law of motion has no hierarchical relationship with other attributes.

In this study, we classified misconceptions about force and motion into six groups. The categorization differs from that by Hestenes et al. (1992). They grouped the misconceptions into six groups: 1) kinematics, 2) impetus (or Newton's first law of motion), 3) active force (or Newton's second law of motion), 4) action/reaction pair of force (or Newton's third law of motion), 5) concatenation of influences (or superposition principle), and 6) other influences on motion (i.e., resistance and gravity). The kinematics category was excluded from grouping in this study because the concept describes the motion of objects without considering the forces that cause them to move, which is essential to learning the force and motion concept. The exclusion was also to ensure it is relevant to the study of the force and motion. Additionally, the frictional force and the gravitational force categories were separated from categories of forces in the taxonomy of misconceptions proposed by Hestenes et al. (1992), in order to give detailed information about diagnosis of learners' misconceptions.

We determined the hierarchical relationships among attributes or categories to construct the cognitive model of force and motion, which was validated by the experts on the subject matter. The model gives information to instructors so that they are aware of learners' proficiency in the concept of force and motion. That is, learners must correctly understand the concepts of resultant force and Newton's first law of motion in order to correctly understand Newton's second law of motion. Then, learners will correctly understand the concepts of frictional force and gravitational force. Additionally, there are no hierarchical relationships between misconceptions about Newton's third law of motion and five other categories.

Preliminary results in this study led to the construction of a Q-matrix and diagnostic tests that were integrated with the CDA. Therefore, future studies should develop the Q-matrix based on information from the cognitive model as a guideline for creating test items. The diagnostic tests should be constructed in a constructed-response format because learners are able to give answers in their own words, which provide abundant information to diagnose misconceptions (Ma & de la Torre, 2016).

After constructing test items according to the Q-matrix, this matrix will be validated by experts on the subject matter. That is, the experts code each item for attributes assessed the given item. Next, the items will be piloted with small samples using think-aloud protocols to gather data used for validating the cognitive model and the Q-matrix. Then, the CDM will be employed to analyze learners' responses from the test to estimate the profile of the mastery of attributes for each individual learner and item parameters and to identify potential misspecifications of the Q-matrix (Lee & Sawaki, 2009). The CDMs are used to diagnose misconceptions, such as the bug deterministic inputs, noisy or gate model (Bug-DINO) (Kuo et al., 2016), and the scaling individuals and classifying misconceptions model (SICM) (Bradshaw & Templin, 2014). Finally, diagnostic results will be reported to learners and stakeholders, so they can design and create remedial programs.

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