Primary School Teachers' Perceptions of the Bar Model Method in Mathematical Problem Solving: The Case of Border Patrol Police School Teachers

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Abstract

Two online workshops were conducted to introduce the bar model method as a problem-solving heuristic to the Thai Border Patrol Police (BPP) officers who also served as primary mathematics teachers in schools located in remote areas. These security personnel had limited teaching experiences. Therefore, teaching their students to solve mathematical word problems, a difficult topic for many students, is very challenging. This paper seeks to learn the effectiveness of online workshops and the BPP teachers' receptiveness to the bar model as a problem-solving heuristic. An online questionnaire was used to ascertain 333 participating teachers' perceptions towards the bar model. Teachers expressed positive perceptions towards the use of the bar model. Most of them reflected that they could solve mathematical word problems using the bar model method after the workshops. They felt the bar model could support their students' problem-solving ability as a useful tool for visualizing and understanding word problems. Although the teachers would like to implement the bar model in their teaching, they had certain reservations such as their own understanding of the method and the students' language barriers. These issues may increase the challenges teachers face while teaching and learning the bar model. Teachers requested for additional training courses to review the bar model method, and other supplementary supporting teaching and learning resources such as lesson plans, and student worksheets. The paper concluded by offering some recommendations concerning the future design of professional learning to support BBP teachers use of the bar model in their teaching.

Keyword: Mathematics Teachers; Problem Solving; Bar Model; Model Method; Remote Education

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การรับรู้ของครูระดับประถมศึกษาในโรงเรียนตำรวจตระเวนชายแดนต่อการใช้บาร์โมเดลในการแก้ปัญหาทางคณิตศาสตร์

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บทคัดย่อ

การอบรมเชิงปฏิบัติการรูปแบบออนไลน์เพื่อแนะนำการใช้บาร์โมเดล ซึ่งเป็นยุทธวิธีในการแก้ปัญหาทางคณิตศาสตร์ได้จัดขึ้น 2 ครั้ง ให้กับครูคณิตศาสตร์ระดับประถมศึกษาในโรงเรียนตำรวจตระเวนชายแดน (ตชด.) ที่อยู่ในพื้นที่ห่างไกล ซึ่งครู ตชด. เหล่านี้มีประสบการณ์การสอนที่จำกัด ดังนั้น การจัดการเรียนรู้เรื่องการแก้ปัญหาทางคณิตศาสตร์ซึ่งเป็นหัวข้อที่ยากสำหรับนักเรียนหลายคนจึงเป็นงานที่ท้าทาย งานวิจัยนี้มุ่งเน้นที่ศึกษาประสิทธิผลของการจัดการอบรมเชิงปฏิบัติการในรูปแบบออนไลน์และการรับรู้ของครู ตชด. ที่มีต่อการใช้บาร์โมเดลเพื่อเป็นยุทธวิธีในการแก้ปัญหา แบบสอบถามรูปแบบออนไลน์ถูกใช้ในการเก็บรวบรวมข้อมูลจากครูที่เข้าร่วมในงานวิจัย 333 คน หลังจากการอบรม ผลแสดงให้เห็นว่าครูมีทัศนคติเชิงบวกต่อการใช้บาร์โมเดล ครูส่วนใหญ่สะท้อนว่าพวกเขาสามารถใช้บาร์โมเดลในการแก้ปัญหาด้วยตนเองได้ และคิดว่าบาร์โมเดลสามารถใช้ส่งเสริมความสามารถในการแก้ปัญหาของนักเรียน เป็นเครื่องมือที่มีประโยชน์ในการทำให้เห็นภาพและทำความเข้าใจโจทย์ปัญหา ถึงแม้ว่าครูส่วนใหญ่ต้องการนำบาร์โมเดลไปใช้ในการจัดการเรียนรู้แต่ก็มองเห็นว่าอาจมีข้อจำกัดบางประการ เช่น ความเข้าใจในวิธีการใช้บาร์โมเดลของครู และอุปสรรคทางภาษาของนักเรียน ซึ่งข้อจำกัดเหล่านี้เป็นความท้าทายที่ครูต้องเผชิญ นอกจากนี้ ครูสะท้อนถึงความต้องการเกี่ยวกับหลักสูตรการอบรมเพิ่มเติมเพื่อทบทวนวิธีการใช้บาร์โมเดล รวมทั้งทรัพยากรและสื่อเพิ่มเติมอื่น ๆ เช่น แผนการสอนและใบงานของนักเรียน บทความนี้มีการสรุปผลและให้ข้อเสนอแนะเกี่ยวกับการออกแบบการเรียนรู้เพื่อสนับสนุนการใช้บาร์โมเดลของครู ตชด. ต่อไปในอนาคต

คำสำคัญ: ครูคณิตศาสตร์, การแก้ปัญหา; บาร์โมเดล; โมเดลเมธอด; การศึกษาทางไกล

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Introduction

Although mathematical problem solving is a core part of the Thai mathematics curriculum, yet many Thai students are challenged by such tasks. Phonapichat, Wongwanich, and Sujiva (2014) found that Grade 5 and 6 students had difficulties (i) in understanding keywords in the problem text, thus they could not interpret the meaning of problems in mathematical sentences, and (ii) assessing relevant information needed to solve a problem. Students (iii) tended to guess the answer without any thinking process whenever they do not understand the problem, (iv) were impatient and did not like to read mathematical problems, and (v) did not like to read word problems that were too lengthy. These results highlighted the importance of understanding the problems. If a student could not understand the problem, they were unlikely to engage with the related thinking process needed to solve the problem.

The Border Patrol Police (BPP) schools, located at the Thai borders, play multiple roles in modernizing and integrating minorities, including ethnic groups and people near the border of Thailand (Hyun, 2014). Many students come from different ethnic groups within Thailand or from neighboring countries, such as Myanmar. Thai language is not the lingua franca of these groups. Hence, it is extremely difficult for these groups of students to understand mathematical word problems; often not enough to solve them.

Many of the BPP schools lack teaching resources. In terms of human resource, all teachers in the BPP schools are officers of the Border Patrol Police. Thus, the majority of the BPP teachers are without the appropriate teaching qualifications. If primary mathematics teachers with the appropriate teaching qualifications are already challenged when teaching problem solving, BPP teachers stationed in remote areas, facing even more challenges than their peers teaching at conventional Thai schools, particularly in mathematics and science (Chairhit, 2021), as they learned the pedagogy to teach students. Therefore, it is crucial to offer sufficient supports, such as training courses, to the BBP teachers, particularly focusing on teaching techniques related to solving mathematical word problems. This will enable them to teach their students effectively.

The model method or bar model is one of the heuristics can be used to help students solving mathematical word problems (Ng & Lee, 2005; 2009). In Thailand, the bar model was introduced and presented in many series of mathematical student textbooks as a tool for solving word problems. Several studies investigated whether the bar model could support primary school students' ability to solve word problems, in terms of increasing students' learning achievement and the accuracy of solving word problems. Results confirmed that the bar model method was a useful tool to help Thai

students solving word problems more successfully. Because of the bar model's effective graphical supports, this method might help students with learning difficulties or language barriers to visualize, understand, and solve word problems more effectively.

To ensure the quality of teaching the bar model method in classrooms, teachers must be familiar with this method. Training teachers to use the bar model and understand ways to implement it into the classroom is critical. Moreover, teachers' perception might affect or influence their teaching. Teacher's perception among different groups of teachers also can be different e.g. novices versus experts, or urban school versus rural school teachers. Therefore, this present study aimed to explore the Thai BPP teachers' perceptions towards the use of the bar model as a tool for mathematical problem solving, introduced during a workshop.

Objectives

Objectives of the present study were as following:

1. To explore the effectiveness of bar model workshops in terms of the BPP teachers' attitude towards the use of the bar model after a workshop

2. To investigate the BPP teachers' receptiveness to the bar model as a problem solving heuristic.

Literature Reviews

The model method, or bar model, is one of the heuristics can be used to help students to solve mathematical word problems. In 1983, the Singapore Ministry of Education officially introduced a problem solving heuristic, the model method or bar model, into the primary mathematics curriculum. It was believed that providing children with the means to visualize a word problem would make the underpinning structure of the problem overt. Once children understood the structure of the problem, they were more likely to solve it. (Ng & Lee, 2005; 2009). Before the model method's introduction in 1981, most Singapore primary school students could only solve word problems when keywords such as 'altogether' and 'left' were provided; they struggled to solve word problems without these cues (MOE, 1981, as cited in Kaur, 2019). But, with the introduction of the model method as a problem-solving heuristic, Singapore became top at Trends in International Mathematics and Science Study (TIMSS) in 1995 (Dindyal, 2006).

There are three types of the bar models, with (i) part-whole model, (ii) comparison model, and (iii) change model or before-after model (Kaur, 2019; Kho, Yeo, & Lim, 2009). Each of these models serve a distinct purpose. The part-whole model helps students

work through word-problems involving relationships between the whole and its parts. The comparison model is used to compare quantities. The change model or before-after model shows the relationship between the new value of a quantity and its original value after an increase or a decrease. The bar model is intended as a tool for solving arithmetic and algebra word problems involving whole numbers, fractions, ratios, and percent.

Many studies carried out on the efficacy of the model method. For example, Osman et al.'s study (2018) reported that primary school students in Malaysia had improved their level of performance in solving non-routine word problems involving addition and subtraction when using the bar model. The study of de Koning, Boonen, Jongerling, van Wesel, & van der Schoot, (2022) suggested effective graphical supports of the bar model method in solving the 'compare' problems – word problems involving comparison of quantities – especially those with inconsistent language, which requires students to derive the correct relation from the text. Results suggested that the bar model has helped students to better articulate their understanding of word problems and to improve their problem-solving performance. Primary school students with mathematics learning difficulties also have benefited from the use of a bar model drawing strategy. After learning the bar model students increased their accuracy in solving word problems and their ability to use cognitive strategies to solve the problems (Morin, Watson, Hester, & Raver, 2017; Preston, 2016).

Some studies reported on student's difficulties in the use of bar models. Poh's study (2007) reported that while solving word problems using the bar models Singapore Primary 3 students had difficulties in: (i) understanding the problem, especially those expressed in inconsistent language, (ii) using the model to determine the solution steps, and (iii) deciding what the correct model should be. (iv) Students also showed a lack of understanding of the syntax of mathematical language. The bar model might not always be the method of choice if students had alternative ways to solve the problems. Similarly, Goh's study (2009) noted some limitations that affected Singapore Primary 5 students' success while using the bar model to solve 'before-after' problems. Errors during transformation, like drawing incorrect bar models, might come from students' incomprehension of the problems or lack of skills in transforming the bar models, especially for multiple-step word problems. Students might have difficulties during the translation of the 'visual' of the bar models to mathematical sentences because of their lack of skills. These findings aligned with results from Ng and Lee's study (2009) that student's mistakes while using the bar model to solve algebraic word problems were more a result of erroneous representations of the problems than related to computational mistakes. An error in the use of a model can lead to an inaccurate answer. It could be the consequence of incorrectly interpreting data that was accurately represented in

the model, of misrepresenting data, or changing the generator midway through the solution.

There have been a few attempts to study teachers' perception of bar model method. Ng and Lee (2009) interviewed four heads of department (HODs) and 14 Primary 5 mathematics teachers about their opinion towards using model method to solve algebraic word problems. All teachers agreed that the visual and concrete nature of the model method made it a useful problem-solving tool. Five expert mathematics teachers from Kaur's study (2019), who had responsibilities in developing the competence of fellow teachers in the teaching and learning of mathematics in primary schools in Singapore, agreed that the model method was very useful for students to solve whole number arithmetic word problems. They felt the model method helped students who had difficulties in solving word problems to represent and visualize relationships between the givens and goals of word problems. They pointed that students often rely on the bar model to solve compare problems. When the students saw 'more than' or 'less than' conditions in problems, they would draw bar models of the comparison type. Also, students seemed to find the model method very helpful in many topics including fractions, ratio, and percentages. Primary 4 students and onwards tend to draw bar models when solving complex word problems. However, little research outside Singapore is available focusing on teacher's perceptions on the adoption of the model method.

Methods

Bar model workshops

Most BPP schools are located in remote areas, hence two one-day long online workshops using ZOOM was used to introduce the bar model method and to encourage BBP teachers to implement it into their classrooms. Workshop 1 was for BPP Primary 1 to Primary 3 mathematics teachers. Workshop 2 was for BPP Primary 4 to Primary 6 mathematics teachers. Trainers and participants also used group chat of the "Line" application to communicate with each other. Each workshop comprised two parts:

Part I (4 hours): The bar model method was introduced to the BPP teachers. Teachers learned to solve word problems using the three types of bar model, (i) part-whole model, (ii) comparison model, and (iii) change model or before-after model. They learned to solve word problems in the topics related to their grade levels such as addition, subtraction, multiplication, and division of whole numbers, fraction, decimal, percent, and ratio. In each topic, the trainer demonstrated how to ask questions and how to solve mathematics word problems using the bar model. Teachers

solved word problems using bar models and submitted their work via group chat of the "Line" application. Some teachers were asked to present their ideas, which then were discussed together.

Part II (2 hours): BPP teachers were grouped according to grade levels. To deepen their understandings of the bar model heuristic, teachers were given opportunities to solve a set of word problems related to their grades. They also had opportunities to seek clarifications when they had doubts; they shared experiences and discussed how to implement the bar model into their teaching.

Data Collection

An online questionnaire (Google Forms) was developed to gather data from participants. It was sent to all the BPP teachers who attended the bar model workshops. The online survey method was employed in the present study because it was convenient for the participants to answer. The questionnaire comprised both closed and open-ended items (Cohen, Manion, & Morrison, 2017; Creswell, 2014). Three experts in mathematics education checked the validity of the questionnaire. Feedback from the experts were used to improve the quality of questionnaire. The questionnaire was piloted with two primary mathematics teachers. A brief interview was also conducted to check the reliability of the questionnaire. Further amendments were made to improve the clarity of certain items.

Participants

Three hundred and thirty-three teachers participated in the study. Table 1 provides the number of years of experience of the participants.

Table 1   Experience in Teaching Mathematics

Teaching Experience	Less than 3 years	3 - 10 years	10 or more years
No. of teacher (%)	120 (36)	167 (50)	46 (14)

Twenty (6%) participants reported that they often or always taught students to use the bar model to solve word problems in their classes. One hundred and twenty-one (36%) participants reported that they sometimes or rarely taught students to use the bar model. One hundred and ninety-two (58%) teachers had not taught students to use the bar model.

Data Analysis

Data analysis was conducted by descriptive statistics, and content analysis. Data from closed-ended questions were analyzed using descriptive statistics, whereas data from open-ended questions were analyzed using content analysis.

Results

Results are reported in two parts. Part 1 reports on the findings related to the effectiveness of the bar model workshops introducing the bar model method in terms of the BPP teachers' attitude toward the use of the bar model after the workshops. Part 2 reports the findings related to the BPP teachers' receptiveness to the bar model as a problem-solving heuristic, i.e., perceived advantages and difficulties, and needs of supports.

Part 1: Effectiveness of the Bar Model workshops

a) Reflection on teacher's ability to use the bar model

The closed-ended responses of the BPP teachers' about their ability to use the bar model method are summarized in Table 2.

Table 2   Frequencies and Percent of Responses to Statement about Ability to Use the Bar Model Method to Solve Problems

Item	Frequency (%)				
	Strongly disagree	Disagree	Undecided	Agree	Strongly agree
1. I like using the bar model to solve word problems.	1 (0.3)	2 (0.6)	43 (12.9)	170 (51.1)	117 (35.1)
2. Using the bar model to solve word problems is difficult for me.	85 (25.5)	98 (29.4)	73 (21.9)	63 (18.9)	14 (4.2)
3. I can solve word problems using the bar model.	1 (0.3)	10 (3)	48 (14.4)	192 (57.7)	82 (24.6)

Responses suggest that the teachers liked to use the bar model method (Item 1, 86.2% Agree or Strongly Agree). They thought they could solve word problems using the bar model (Item 3, 82.3% Agree or Strongly Agree). In contrast, teachers disagreed that using the bar model to solve word problems was difficult (Item 2, 3.3% Disagree or Strongly Disagree). Results illustrate that the BPP teachers have a positive attitude toward the bar model method. Most of the teachers understood and had confidence in using the bar model as a tool to solve word problems after attending the bar model workshops.

b) Reflection on teaching the bar model method in classrooms

Teachers' responses to statements about using the bar model method in their classrooms are summarized in Table 3.

Table 3 Frequencies and Percent of Responses to Statement about Teaching the Bar Model in Classrooms

Item	Frequency (%)				
	Strongly disagree	Disagree	Undecided	Agree	Strongly agree
4. I think that I can teach my students to solve word problems using the bar model.	1 (0.3)	4 (1.2)	43 (12.9)	215 (64.6)	70 (21)
5. I think that teaching students to solve word problems using the bar model in classrooms is easy.	0 (0)	5 (1.5)	83 (24.9)	186 (55.9)	59 (17.7)
6. I will implement the bar model in teaching solving word problems in my classroom.	0 (0)	2 (0.6)	38 (11.4)	177 (53.2)	116 (34.8)
7. I will teach solving word problems using different types of bar models to my students.	0 (0)	2 (0.6)	40 (12)	190 (57)	101 (30.3)

Teachers reflected that they could teach the students to use the bar model (Item 4, 85.6% Agree or Strongly Agree) and were willing to implement the bar model in their classrooms (Item 6, 88% Agree or Strongly Agree). They would like to teach students solving word problems using different types of bar models (Item 7, 88% Agree or Strongly Agree). Most of the teachers felt that teaching students to solve word problems using the bar model was easy (Item 5, 73.6% Agree or Strongly Agree). However, a substantial proportion of teachers was unsure whether it was easy for their students (Item 5, 24.9% Undecided). In the aspect of implementation, responses showed that the BPP teachers were convinced and quite confident in teaching their students using the bar model method, after attending the bar model workshops.

Part 2: Teachers' receptiveness to the bar model as a problem-solving heuristic

The closed-ended responses about the use of the bar model as a problem solving-heuristic are summarized in Table 4. Responses indicated that the teachers were convinced of the advantages of the bar model method in helping students to solve word problems (Item 8, 88.3% Agree or Strongly Agree) and in helping students interested in solving word problems (Item 9, 87.1% Agree or Strongly Agree). They felt their students would like to use the bar model method. (Item 13, 83.8% Agree or Strongly Agree).

BPP teachers disagreed that the bar model method was not suitable for their students (Item 10, 65.5% Disagree or Strongly Disagree). They thought the bar model

method was not difficult for their students (Item 11, 45.9% Disagree or Strongly Disagree). However, a substantial proportion of teachers was unsure whether it was difficult for their students (Item 11, 27.3% Undecided). Similarly, while most of the teachers thought the bar model seemed to be helpful for their students (Item 12, 52.6% Disagree or Strongly Disagree), there was uncertainty as to whether their students could solve word problems well without using the bar model (Item 12, 33.3% Undecided).

Table 4   Frequencies and Percent of Responses to Statement about the Use of the Bar Model as a Problem Solving-Heuristic

Item	Frequency (%)				
	Strongly disagree	Disagree	Undecided	Agree	Strongly agree
8. I think that using the bar model can help students to solve word problems easier.	1 (0.3)	5 (1.5)	33 (9.9)	162 (48.6)	132 (39.6)
9. I think that using the bar model can make students interested in solving word problems.	0 (0)	4 (1.2)	39 (11.7)	179 (53.8)	111 (33.3)
10. I think that the bar model is not suitable for my students.	133 (39.9)	85(25.5)	56 (16.8)	44 (13.2)	15 (4.5)
11. I think that using the bar model to solve word problems is difficult for my students.	73 (21.9)	80 (24)	91 (27.3)	73 (21.9)	16 (4.8)
12. I think that my students can solve word problems well without using the bar model.	63 (18.9)	112 (33.6)	111 (33.3)	35 (10.5)	12 (3.6)
13. I think that if I teach my students the bar model, they will like to use this method.	2 (0.6)	5 (1.5)	47 (14.1)	192 (57.7)	87 (26.1)

Furthermore, the open-ended responses were used to triangulate the data from the close-ended responses. Findings were reported as following.

a) Perceived advantages of using the bar model method

Three hundred and eleven BPP teachers (93%) answered to the question, "What do you think are the advantages of teaching and learning using the bar model to solve problems? Why?" The perceived advantages of teaching and learning using the bar model method were analyzed and identified into six themes.

Tools for understanding math word problems (Code: A1). Most teachers perceived that the bar model was a tool to help students to understand the problems

(163 respondents). Their responses included phases or words such as "understand problem easier" or "better understanding".

Tools for visualizing math word problems (Code: A2). Teachers thought the bar model made the problems easier to visualize especially while comparing quantities (151 respondents).

Tools for finding solutions (Code: A3). Teachers felt that the bar model would help students to find solutions easier or faster (46 respondents). Here are some of the respondents' statements:

"It (bar model) makes students understand better about the problems. Some students may be still confused with the problem that cannot be clearly analyzed. If the bar model is used in teaching, it will make students easier to understand and visualize." (A1, A2)

"It (bar model) presents in concrete. Children are easy to understand because by looking at the picture it becomes more easily understood. Then, (children) can solve the problems" (A1, A2, A3)

Increasing student's interest (Code: A4). Teachers thought that using the bar model would increase student's interest or attention in learning (12 respondents). Here are some of the respondents' statements:

"Students will enjoy learning because it's easy to understand." (A1, A4)

"(The advantage is) using pictures to help you think. Because of visualizing and having pictures involved, it makes learning more exciting." (A2, A4)

Supporting systematic thinking (Code: A5). The BBP teachers perceived that the bar model supported students' systematic thinking. The bar model method showed a clearer process to the students (9 respondents), and as one respondent stated:

"(From the Bar Model,) children will know the steps to solve the problem, where to go and where the results come from." (A5)

Reduced students' difficulties (Code: A6). Some teachers noted (7 respondents) the advantages of using the bar model for reducing students' difficulties while solving word problems. For example, they stated that the bar model could support students who could not read Thai language well. The bar model reduced the difficulty of solving complex problems or longer statements, and it was better to draw a bar instead of drawing objects one by one.

b) Perceived difficulties of using the bar model method

Two hundred and twenty-nine teachers (69%) provided answers related to the difficulties they perceived if they implemented the bar model method in the BPP school contexts. Responses were analyzed and identified into seven themes.

Student's readiness (Code: D1). Most teachers (90 respondents) revealed their concerns about student's readiness. Students' ability to read, write, and understand texts in word problems could be one of the barriers, for example, for minority or ethnic groups of students. The command of Thai language there was always a language barrier, including communication between teachers and students. Here are some of the respondents' statements on this issue:

"Some of the students couldn't read and write. I'm not sure how much the bar model will make them understand." (D1)

"Ethnic groups of students do not understand the meaning of Thai language. It is difficult to communicate." (D1)

Teachers were also concerned about the students' low performance in mathematics and lack of mathematical skills, especially in computation. They also noted the diversity of students in a class, for example, slow learners might not be able to understand and use the bar model. Some teachers questioned that students at Primary 1 might be too young to learn the bar model. Here are some of the respondents' statements:

"Students who are not ready or have learning disabilities might find it difficult to understand." (D1)

"(The bar model is) not suitable for students who are not good at math because it's difficult for them to figure out what they have to do and then have to draw a picture. (D1)

Teacher's readiness (Code: D2). Many teachers (79 respondents) noted their concerns on teacher's readiness. The bar model method was new to both teachers and students, so its implementation was a challenge for teachers at the beginning. They were also concerned about teachers' expertise and their understanding of the bar model method, particularly how to make students understand the method. Here are some of the respondents' statements on this issue:

"Teachers are still new (to the bar model). I'm still confused about the bar model. I have to practice it often." (D2)

"The teacher had to come up with an easy bar model so that the children could follow along and find the correct answer." (D2)

Types of word problems (Code: D3). Many teachers (59 respondents) felt that some types of word problems might not be suitable for the bar model method. They thought complex or complicated word problems such as 2-steps or multi-steps word problems were quite difficult to be used with the bar model method. This included also word problems with large numbers. Here are some of the respondents' statements:

"In case the problem is complex and has many steps, it is difficult to draw bar models understandable." (D3)

"For a complicated question, there are some points that do not require the bar model. The students may be confused that they must use bar model every time" (D3)

Time consuming (Code: D4). Teachers (24 respondents) shared their concerns about time usage when applying the bar model, as one of the respondents answered, "Time for teaching, making students to understand".

Lack of teaching materials (Code: D5). Teachers (16 respondents) stated about the lack of teaching and learning materials in their schools, which might hinder the teaching and learning efforts. However, most of the responses did not clearly state the materials they needed for teaching the bar model.

Difficulty of the bar model method (Code: D6). Teachers (13 respondents) stated about the difficulty of the bar model method itself. To them, the bar model method was difficult to use. For example, they thought it was difficult to imagine and draw the bars, as there were many ways of thinking and solving, which might lead to students' confusion. They were concerned that is was difficult for students to transfer from pictures to symbols, as one respondent stated:

"Drawing pictures (bar model) to visualize a word problem. If students do not understand the picture, they will not be able to solve the problem." (D6) (Teacher 18)

Others (Code: D7). One teacher questioned about the design of the student exercise book; it might provide space for drawing the bar model. Another teacher pointed out the mismatch between scoring method for written word problems task in the National Test (Mathematics) and the bar model method.

c) Need for supports

Eighty-three teachers (24.9%) suggested they needed further in-service support after the bar model workshops. Possible supports were: (i) additional training courses, (ii) learning resources for reviewing what they learned from the bar model workshops, (iii) lesson plans, and (iv) teaching and learning supplementary and materials, e.g., worksheets, exercises, bar model templates.

Discussion

Effectiveness of the Bar Model workshops

Results show that the BPP teachers became familiar and had confident in their use of the bar model method after attending the bar model workshops. Although each workshop was a 1-day online workshop with many limitations, the BBP showed that they

understood how to use the bar model as a problem-solving heuristic. They perceived the bar model in a good way, as a useful heuristic helping students solving mathematical word problem. In addition, most teachers were interested and willing to adopt the bar model method into their teaching. To them, it seemed possible to implement the bar model in the BPP school context. It might be the effect of the bar model workshop that required participants to solve word problems themselves, then share their work in group chat, and further had opportunities to present their ideas with peers verbally via Zoom. The workshop promoted learning communities. Moreover, teachers had a chance to discuss the bar model with trainers and peers who were teaching at same levels. It might have helped teachers to clear their confusion as well as aspire them to use the bar model method in their lessons. In short, these online workshops appear to be successful in acquainting the BPP teachers with the bar model method and fostering their positive attitude toward this method. However, when it comes to implementing it in real classroom settings, relying on these workshops for learning not be sufficient for them.

Teachers' receptiveness to the bar model as a problem-solving heuristic

BPP teachers perceived both advantages and difficulties of the use of the bar model. In terms of advantages of the bar model method, BPP teachers thought using the models could support their students to understand word problems easier than only reading from texts. Drawing bar models would help students to visualize the problems and lead to a better understanding. The use of the bar model method could enhance students' ability to solve word problems. Results aligned with outcomes from the questionnaire and findings of Ng and Lee' study (2009) and Kaur's study (2019); Singapore teachers felt that the visual and concrete nature of the bar model made this method a useful problem-solving tool. BPP teachers thought the bar model could increase students' interest in solving word problems and promote student's systematic thinking; these points seemed different from the case of Singapore teachers. The reason might be the differences between their experiences in this heuristic. Since the bar model was new to many BPP teachers, they might think it was very interesting for their students. Moreover, teachers reflected that the bar model might reduce their students' difficulties in solving word problems, particularly for low achievement in mathematics or students with language barriers.

While most teachers noted the usefulness of the bar model and would like to implement it into their classrooms, they also pointed out many concerns related to the implementation process. They were concerned about student's readiness (e.g., ability to read and comprehend the texts, students' math skills) and teacher's understanding of the bar model. Teachers were concerned about the usefulness of the bar model with

complex or complicated word problems such as multi-step word problems. This issue seemed to be different from the findings of Kaur's study (2019); Singapore expert teachers found that the bar model was very helpful in topics of fractions, ratio, and percentages, as well as in solving complex word problems. BPP teacher might think differently from Singapore experts because of their difference in teaching experiences. Teachers highlighted other constraints that could affect the implementation of the bar model. Teaching the bar model could be time consuming. Also, there was a lack of teaching materials, the design of student worksheets, national assessment, and other issues. Because the bar model method was new to them, some teachers reflected that the bar model might be difficult for their students; e.g., to imagine and draw the bars, or to transfer from picture to symbols.

BPP teachers indicated what possible support they might need, e.g., additional training courses, learning resources, and teaching and learning materials. This aligns with constraints and difficulties they described, especially their concerns about teachers' understanding of the bar model. It also shows that learning from a 1-day bar model workshop was not enough for the BPP teachers. They might understand how to use the bar model as a problem solving heuristic, but it is not enough for them to implement a new teaching method into their class with confidence. It echoes findings from previous research, where teacher professional development programs modeling effective practice and invite teachers to try them out tend to be more successful than programs that devote resources primarily to changing attitudes first (Ingvarson, Meiers, & Beavis, 2005). During the implementation period, teachers also need support such as follow-up from trainers (Ingvarson et al., 2005).

Conclusions

This paper seeks to understand the effectiveness of online workshops and Border Patrol Police (BPP) primary school teachers' receptiveness to the bar model method as a problem-solving heuristic, after attending bar model workshops. Findings from questionnaires revealed that teachers became confident and had positive perceptions and attitudes towards the use of the bar model method. BPP teachers reflected that they liked and understood solving problems using the bar model method. Most teachers agreed that they would like to implement the bar models in their classrooms. The bar model was useful as a tool for visualizing and understanding problems. This method might be able to support their student mathematical problem-solving abilities. Constraints and difficulties in the implementation in the BPP school contexts are teachers and student's readiness, e.g., students' ability in mathematics and

command in language, as well as teachers' understanding of the bar model method. This study offers avenues to help Thai BPP teachers to use aids to improve their students' problem-solving activities. It is therefore important to provide these teachers the moral and educational supports to continue their work with the students in remote places.

Recommendations

Findings here suggest that the online workshops to introduce the bar model method might help some teachers to gain confidence and positive perceptions on the use of the bar model as a problem solving tool. The online workshop utilizing Zoom and Line applications can be an alternative method for training teachers in remote areas. However, teachers still need other support after the workshops. The future design of professional learning to support the teachers use of the bar model in their teaching should include continuous supports from educators (e.g., follow-up meetings or additional workshops) and learning communities for them to ask questions or share ideas related to the implementation of the bar model.

This study explored the BPP teachers' perception toward the bar model method after attending bar model workshops. To generalize the research findings, further work can compare the effects of this workshop with other workshops as well as expand the samples with other groups of teachers. Further research can incorporate a control group or a pre-and post-test design to enhance the robustness of the findings. Further studies might also investigate teachers' teaching abilities and practices and their perceptions on their use of the bar models in their actual teaching practices.

Acknowledgments

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